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**Olomoucian
Days
of
Applied
Mathematics**

Book of Abstracts

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**Department of Mathematical Analysis
and Applications of Mathematics
Faculty of Science
Palacký University in Olomouc**

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Preface

The conference “Olomoucian Days of Applied Mathematics 2013” (ODAM 2013) is held as a part of the official celebrations at the 440th anniversary of founding the Palacký University in Olomouc. The Palacký University is the oldest university in Moravia and the second oldest in the Czech Republic (second only to the Charles University in Prague). At present it is a modern educational institution with a broad range of both study programs and research activities. More than 23 000 students are enrolled in its eight faculties, which accounts for no less than one-fifth of the population of the city of Olomouc.

The foundation of the O DAM conferences was laid in 1999 by Prof Lubomír Kubáček, an outstanding figure in Czech and Slovak mathematical statistics, the then Head of the Department of Mathematical Analysis and Applications of Mathematics. He took up the tradition of seminars in applied mathematics held at this department and established a tradition of friendly meetings of applied mathematicians, alternately focused on mathematical statistics and fuzzy sets on the one hand, and numerical mathematics on the other.

The financial support for the project “Streamlining the Applied Mathematics Studies at Faculty of Science of Palacký University in Olomouc” (MAPLIMAT for short) made it possible, in 2011 for the first time, to organize the conference O DAM 2011 as a full-scale international conference. It was the organizers’ pleasure to hold the conference right on the occasion of the important anniversary in the life of Prof Kubáček who has contributed so much to the development of applied mathematics at the Faculty of Science.

The MAPLIMAT project, run under the operational program Education for Competitiveness and subsidized from the European Social Fund and the Czech State Budget, significantly contributed to enhancement of applied mathematical studies at Faculty of Science of Palacký University. Over the three years of its course, 5 study programs have been innovated (bachelor to post-doctoral levels), 54 e-learning study supports have been created in LMS Moodle, 16 internal textbooks published, a new computer room opened, bachelor- and master-level company-based scholarships established, postgraduate international collaboration strengthened. Among the benefits of the MAPLIMAT project we also subsume an increased interest of students in research activities that has been stimulated by their attendance at lectures given by foreign experts under this project, and especially at the O DAM 2011 conference.

Our team of MAPLIMAT workers have therefore decided to use the remaining funds in this project to organize the O DAM 2013 conference, again on an international level, instead of the originally planned concluding conference of the project. (One section of O DAM conference will be devoted to the results of MAPLIMAT.)

We are very pleased by the fact that so many leading experts from different areas of applied mathematics accepted our invitation to participate in O DAM 2013—coming from Finland, Italy, Austria, Germany, Spain, Switzerland, USA, Slovakia and, of course, Czech Republic. It is a great honor that as a keynote speaker of the O DAM 2013 conference we will be able to welcome here at Palacký University Prof Christer Carlsson, President of the International Fuzzy Sets and Systems Association (IFSA).

It is our hope that, same as in 2011, the ODAM conference will provide an opportunity to acquire new knowledge, make new professional contacts, meet old friends and start new friendships. On behalf of all the organizers I wish to all participants of ODAM 2013 days pleasantly spent in our beautiful historical city of Olomouc!

To conclude, I would like to express my thanks to all team workers and all student assistants for their help before and during this conference.

Jana Talašová
Chief Manager of MAPLIMAT Project

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Soft Computing in Analytics

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Abstract

In serious decision making we fall back on normative decision theory and the belief that it is possible to find and make optimal decisions. Serious decision making is (for instance) concerned with questions on how to best use capital for interdependent production, logistics and marketing networks when key indicators on return of capital employed should be competitive in industrial comparisons. Real life decision making suffers from uncertainty, imprecision, missing information and knowledge and the lack of a good enough data base (despite advanced IS technology); this has made it hard to justify the use of normative decision theory, analytics models and optimisation algorithms—“we will find optimal solutions to problems that do not exist in real life”. This has been a dilemma for a couple of decades; there is a need for optimal decisions, we have the necessary mathematical tools but we cannot make real life decision making optimal, at least not for industrial scale problems.

Soft computing offers a way out of the dilemma: *at some point there will be a trade-off between precision and relevance; increased precision will be followed by a loss of relevance, and vice versa*. We will show the implications of this insight with some results on fuzzy real option valuation applied to portfolios of projects in the Finnish forest industry. We have replaced parts of the classical real options theory by introducing fuzzy numbers and possibility theory and we show that we will get decision models that are precise enough to produce optimal solutions and relevant enough to make sense for real life decision making.

Key words

Decision making, imprecision, optimal solutions, analytics, soft computing.

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Nonlinear Rescaling Method and Self-Concordant Functions

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Abstract

Nonlinear rescaling (NR) is a tool for solving large-scale nonlinear programming problems. Several issues still remain for research in NR theory. The attention was focused on three of them:

- describe the parameters of the primal-dual nonlinear rescaling method with dynamic scaling parameter update (PDNRD)
- analyse the application of self-concordant functions in NR theory
- give reasons for the choice of the nonlinear rescaling function (both theoretical and practical)

PDNRD method was tested on two quadratic programming problems with quadratic constraints (the chord problem and the steel brick problem). Based on the testing, the conclusions about setting the parameters of PDNRD method were made. It was made out that increasing the number of variables in a problem has not a consequence in the increasing number of solutions of primal-dual system. This fact supports the applicability of PDNRD method on problems of arbitrary size.

Next, convergence and choosing an initial approximation are discussed when minimizing a self-concordant function using Newton's method. Analysis of convergence is known in this case, but the theorem about choosing an initial approximation is a new contribution. The application of self-concordant functions in NR theory is described.

The connection between modified logarithmic barrier function $\psi(t)$ and self-concordant functions is mentioned as a theoretical reason for using $\psi(t)$ as NR function. Instead of function $\psi(t)$, quadratic extrapolation of $\psi(t)$ can be considered. Numerical experiments with different NR functions were made to support practical reasons for using quadratic extrapolation of $\psi(t)$ as NR function.

Key words

Convex optimization, nonlinear rescaling method, self-concordant functions.

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Cox Proportional Regression Model with Delayed Entry and its Application to Mitral Valve Replacement Study

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Abstract

In most clinical studies the patients are observed for extended time periods in order to evaluate one or more interventions (for example, drug treatment, approaches to surgery, etc.) in treatment of a disease, syndrome, or condition. The primary event in these studies is death, relapse, adverse drug reaction, or development of a new disease. The follow-up time may range from few weeks to many years. The longitudinal studies has increased the importance of statistical methods for time-to-event data that can incorporate time-dependent covariates. The Cox proportional hazards model is one such method that is widely used. It is a statistical technique for exploring the relationship between the survival of a patient and several explanatory variables. The main objective of this contribution is to discuss Cox's regression models where right censoring and left truncated survival data are considered. We report the results of simulations and illustrate the method using a survival study. In this retrospective study we compare and analyze results of mitral valve replacement in children under 18 years.

Key words

Survival analysis, Cox model, Mitral valve prosthesis, delayed entry.

References

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Fuzzy Sets and Rough Sets in Prototype-based Clustering Algorithms

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Abstract

Fuzzy sets and rough sets provide an effective means for handling uncertainty in real life situations. The theory of fuzzy sets (Zadeh, 1965) is based on the notion of a membership function on the domain of discourse, assigning to each object a grade of belongingness to an imprecise (ill-defined) concept. The theory of rough sets (Pawlak, 1982) is a tool for describing ambiguity caused by limited discernibility of objects in the domain of discourse. Applications of both theories can be found in virtually any domain.

We will review incorporation of fuzzy sets and rough sets in the classic hard c -means algorithm (McQueen, 1967), which was designed to find a partition of a set of n objects into c clusters with high intra-class similarity and low inter-class similarity. The following algorithms will be discussed: fuzzy c -means (Bezdek, 1981), possibilistic c -means (Krishnapuram and Keller, 1993), rough c -means (Lingras and West, 2004), rough-fuzzy c -means (Mitra, Banka and Pedrycz, 2006) and rough set based generalized fuzzy c -means (Maji and Pal, 2007).

In 1998 Pedrycz proposed the theory of shadowed sets as a conceptual bridge between fuzzy sets and rough sets. We will conclude our review with a short description of the shadowed c -means algorithm (Mitra, Pedrycz and Barman, 2010), which applies the concept of shadowed sets to a modification of a rough-fuzzy c -means clustering algorithm.

Key words

Fuzzy sets, rough sets, shadowed sets, clustering, vague partition, c -means clustering algorithm.

References

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On an Explicit Solution of a Finite Network Problem

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Abstract

A classical problem in electric circuit theory is the computation of the resistance between two nodes in a resistor network. Such network is represented by a nodal conductance matrix \mathbf{L} (Laplacian matrix), which is real. If no external source is connected to the nodes, \mathbf{L} is symmetric. Traditional methods (analysis of the network) enable the computation of the resistances, however, with the growing number of nodes, the number of equations to solve grows very quickly. A new effective approach was introduced in [1]. Due to the first Kirchhoff's law, one of eigenvalues of the Laplacian matrix \mathbf{L} is zero. Therefore, the inverse of \mathbf{L} cannot be considered and the computation of resistances becomes complicated. In [1], this difficulty was circumvented; the resistance $R_{\alpha\beta}$ between two arbitrary nodes α and β in a resistor network with N nodes is expressed in terms of eigenvalues $\lambda_i \neq 0$ and eigenvectors ψ_2, \dots, ψ_N of the symmetric Laplacian matrix \mathbf{L} associated with the network:

$$R_{\alpha\beta} = \sum_{i=2}^N \frac{1}{\lambda_i} |\psi_{i\alpha} - \psi_{i\beta}|^2 \quad (1)$$

During the test development process, an external source is connected to the nodes of the network and the Laplacian matrix can become non-symmetric. If the computation of resistances is performed in a traditional way, it might be rather time consuming: a great amount of simulations during testing requires solve very large systems of equations. An explicit formula similar to (1) can lead to a significant analysis time reducing, since the evaluation of resistance should require only one simulation [2]. But the beautiful Wu's formula (1) applies exclusively, if the Laplacian matrix \mathbf{L} is symmetric. Recently, we have successfully extended (1) on a large family of networks with nonsymmetric Laplacian matrix. The efficiency of the approach using the generalized formula is actually tested at the Faculty of Electrical Engineering and Information Technology STU in Bratislava.

Key words

Two-point resistance, Laplacian matrix, eigenvalues, eigenvectors.

References

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A Univariate Multiple Use Calibration

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Abstract

We consider a univariate linear regression model

$$Y(x_i) = f^T(x_i)\beta + \epsilon_i, \epsilon_i \sim N(0, \sigma^2), \quad i = 1, \dots, n, \quad (1)$$

where $Y(x_i)$ are independent normally distributed observations, $f^T(x_i)$ is a q -dimensional known function of x_i , $i = 1, \dots, n$, β and σ^2 are the unknown parameters of the model. Let $\hat{\beta}$ denote the least squares estimator of β , and let S^2 denote the residual mean square based on $n - q$ degrees of freedom. It is required to construct confidence intervals for the unknown independent values $x_1^*, x_2^*, \dots, x_k^*$ corresponding to K additional observations $Y_{n+1}^*, Y_{n+2}^*, \dots, Y_{n+K}^*$, where K is unknown and possibly arbitrary large. The sequence of confidence intervals will be constructed by using the same estimates of β, σ^2 and will have the property that at least a proportion γ of such intervals will contain the corresponding true x -value with confidence $1 - \alpha$. The multiple use confidence interval is assumed of the form $\{x: f^T(x)\hat{\beta} - \lambda(x)S \leq Y(x) \leq f^T(x)\hat{\beta} + \lambda(x)S\}$ and let $C(x; \hat{\beta}, S)$ denote the coverage of the such an interval, conditionally given $\hat{\beta}, S$. A factor $\lambda(x)$ is to be chosen so as to satisfy the condition

$$P_{\hat{\beta}, S} \left(\frac{1}{K} \sum_{i=1}^K C(x_i; \hat{\beta}, S) \geq \gamma \right) = 1 - \alpha, \quad (2)$$

for every K and for every sequence $\{x_i\}$, see e.g. Krishnamoorthy and Mathew (2009). For a case of a simple linear regression model Mee and Eberhardt (1996) stated that a tolerance interval satisfies (2). The purpose of this contribution is to emphasize that the property of the multiple use confidence interval can be described by the condition (2) in case of a large number of future observations. Based on provided simulation study for a case of a simple linear regression, we recommend to construct the multiple use confidence intervals by using the tolerance interval only in case $K > 300$.

Key words

Multiple use confidence interval, linear regression model.

References

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On Fuzzy Scorecards

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Abstract

Strategic investments require forward-looking analysis and most often face structural uncertainty. This means that precise and detailed information about them is unavailable. Systems used in managing strategic investments must be robust enough to handle the imprecision, while at the same time being simple enough to be easy to use and still smart enough to convey a good overall understanding of the investments. Fuzzy sets [1] and fuzzy numbers are a precise way of representing imprecise information and scorecards, for example the balanced scorecard [2], are a well-known, simple, but structured tool for the collection and analysis of information. A combination of fuzzy numbers and scorecards is a “natural” way to create simple, easy-to-understand, easy to visualize, easy-to-construct, and low-cost tools for the analysis of strategic investments [3]. A simple fuzzy scorecard can function as a basis for a more complex and advanced analyses and analysis systems [4]. Examples of fuzzy scorecards and using them in the real world are given.

Key words

Fuzzy scorecards, structural uncertainty, strategic investments.

References

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Fuzzy Rating or Fuzzy Linguistic?

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Abstract

In a recent paper, Treiblmaier and Filzmoser [3] have discussed the benefits of using a continuous rating scale in contrast to using Likert-type scales in surveys. The comparative analysis indicates that the use of the continuous scale overcome the problems of information loss and also allows for applying advanced robust statistical analyses.

By using a different tool, the mean squared error where the error is based on a versatile metric between fuzzy numbers, the use of “fuzzy continuous” rating scales (actually, a free fuzzy-valued response format questionnaire) has been compared [2] with some fuzzy conversions of Likert-type scales. The comparative study corroborates, with other statistical arguments, the benefits of using the free fuzzy scale in contrast to the fuzzy converted prefixed ones are clear.

This paper aims to show the almost uniform advantage of considering the (free) fuzzy rating scale in contrast to the most usual fuzzy linguistic approaches to the 5-point Likert scales. The comparison will be developed in terms of an index of relative dispersion which has been introduced by Alonso et al. [1]. The drawn conclusion confirms that the mean value is in most of cases more representative for the (free) fuzzy rating than for the fuzzy linguistic approach.

Key words

Fuzzy linguistic, fuzzy rating, statistics.

References

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Thanks

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Calibration Line Problem for Compositions

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Abstract

Calibration is used to extract the linear relationship between the errorless measurement results obtained by measuring the same object on two different measuring methods or devices. We will take the measurement results to be compositional. Compositions require different treatment when one performs standard statistical analysis, like for e.g. the calibration. They are defined as multivariate observations carrying only relative information in their parts, Aitchison (1986). Therefore a proper representation of the compositions seems to be via log-ratios of parts. This representation of the compositions allows us to dismantle the multivariate calibration problem into a $D(D - 1)/2$ partial univariate calibration problems. This means that we can provide calibration for coordinates that are log-ratios of each of the two-part subcompositions. Algorithm for solving the calibration line problem using the linear model approach just like the statistical inference for three-part composition are proposed in Fišerová and Hron (2012). In the contribution we will devote to the task of the calibration problem for compositions in general. Tests for verification of conformity between two methods of measurement will be proposed too. On an example from biochemistry we will demonstrate the usefulness of the theoretical considerations.

Key words

Log-ratio transformations, calibration line, multiple comparisons.

References

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Vector Optimization Results for ℓ -Stable Data

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Abstract

In 2008 the concept of ℓ -stable at a point scalar functions were introduced in [1] as a generalization of $C^{1,1}$ functions—functions with locally Lipschitz derivative. The main aim was to receive more general optimality conditions than for $C^{1,1}$ functions which were extensively studied previously (see e.g. [5]). In subsequent years the attention was devoted to deriving other properties of ℓ -stable at a point functions and to extending ℓ -stability to finite-dimensional spaces in connection with vector optimization ([2, 4]).

I try to summarize the most important of this existing results, especially for the following programming problem:

$$\begin{aligned} &\text{minimize } f(x) \text{ subject to } C \\ &\quad \text{such that } g(x) \in -K, \end{aligned}$$

where $f: \mathbb{R}^N \rightarrow \mathbb{R}^M$ and $g: \mathbb{R}^N \rightarrow \mathbb{R}^P$, $M \in \mathbb{N}$, $N \in \mathbb{N}$, $P \in \mathbb{N}$, are given functions and $C \subset \mathbb{R}^M$, $K \subset \mathbb{R}^P$, are closed, convex, and pointed cones with non-empty interior.

Key words

ℓ -stable function, generalized second-order directional derivative, Dini derivative, weakly efficient minimizer, isolated minimizer.

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Compositional Tables Analysis in Coordinates

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Abstract

Compositional tables as a special case of D -part compositional data [2] represent a continuous counterpart to the well-known contingency tables, which carry relative information about relationship between two (row and column) factors. Their cells as well as in the case of D -part compositions contain quantitatively expressed relative contributions on a whole and for their analysis only ratios between cells are important. This nature of compositional tables requires a specific geometrical treatment, represented by the Aitchison geometry on the simplex. The properties of the Aitchison geometry allow a decomposition of the original table into its independent and interaction parts and consequently investigation of the relationship between factors [4]. The corresponding statistical analysis should be performed in orthonormal coordinates [3]. The resulting isometric logratio (ilr) coordinates form representation of compositional data in the real space and thus all standard statistical methods can be applied. We search for such ilr coordinates that are easy to handle and well-interpretable. The aim of the contribution is to introduce a formula that assigns such coordinates through decomposition of the interaction table into smaller tables; each coordinate then corresponds to odds ratio in these smaller tables [1]. Theoretical results are applied to real-world data from a health survey.

Key words

Compositional data, missing values, rounded zeros, imputation.

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Inconsistency Evaluation in Preference Relations: A Characterization Based on Metrics Induced by a Norm

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Abstract

Assume that an $n \times n$ pairwise comparison matrix \mathbf{A} is represented by a point in the vector space $\mathbb{R}^{n \times n}$ of $n \times n$ real matrices. Then, an inconsistency index of \mathbf{A} can be defined as a distance between \mathbf{A} and the set of consistent matrices, i.e. the minimum distance between \mathbf{A} and a consistent matrix. This approach is already known, but the notion of distance has been proved to be too general to conveniently characterize inconsistency, so that it can lead to unsatisfactory inconsistency indices [3]. In this paper it is proved that a suitable characterization of inconsistency indices is obtained by restricting distances to those induced by norms, under the assumption of additive representation of preferences. Then, an inconsistency index is a seminorm in the linear space \mathcal{L} of skew-symmetric matrices and from this main result several other properties are derived. In particular, the linear space \mathcal{L} can be partitioned into equivalence classes, where each class is an affine subspace. All matrices in an equivalence class share the same value of the inconsistency index, since all of them have the same norm-induced distance from the linear subspace of consistent matrices. Some results due respectively to Barzilai [1] and to Crawford & Williams [2] are extended in a more general framework. It is also proved that norm-based inconsistency indices satisfy a set of five characterizing properties previously introduced, as well as an upper bound property for group preference aggregation.

Key words

Inconsistency index, pairwise comparison matrix, norm, distance.

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A Hybrid Procedure for Network Multi-Criteria Systems

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Abstract

Many of today's systems are characterized by a network structure and evaluation of alternatives is based on multiple criteria. Network systems contain both positive and negative feedbacks. In the paper is proposed a hybrid procedure for operation in such environment. The procedure is based on a combination of DEMATEL, ANP, and PROMETHEE approaches. The DEMATEL method, originated from the Geneva Research Centre of the Battelle Memorial Institute, is especially pragmatic to visualize the structure of complicated causal relationships. DEMATEL is a comprehensive method for building and analyzing a structural model involving causal relationships between complex factors. Analytic Hierarchy Process (AHP), developed by T. Saaty, is a very popular method for setting priorities in hierarchical systems. A variety of feedback processes create complex system behavior. For the network seems to be very appropriate Analytic Network Process (ANP) approach. The ANP makes possible to deal systematically with all kinds of dependence and feedback in the system. PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) methods are method for multi-criteria evaluation of alternatives, based on preference relations. The methods PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed by J. P. Brans and then further developed as a family of methods. A considerable number of successful applications has been treated by the PROMETHEE methodology in various fields. The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use. The combination of these approaches gives a powerful instrument for analyzing network systems by multiple criteria.

Key words

Network problems, multiple criteria, DEMATEL, ANP, PROMETHEE.

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Stepwise Variable Selection in Robust Regression with Compositional Explanatory Variables

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Abstract

Compositional explanatory variables are considered in a multiple linear regression model to explain a non-compositional response. As raw compositional data cannot be used directly in this setting, an approach based on the isometric log-ratio (ilr) transformation is used [1].

A special choice of the ilr transformation allows to test for the contribution of all the relative information concerning one compositional part to explaining the response. This leads to the development of a forward and backward stepwise variable selection algorithm. Using robust regression in combination with a robust Akaike information criterion, results in a robust variable selection procedure among the compositional variables [2]. Simulation results confirm that the method can successfully select good models. In presence of outliers, classical model selection breaks down, while the robust methods still show excellent behavior. The approach is applied to a data set from geochemistry, where those chemical elements that are enriched in the soil due to heavy traffic are identified.

Key words

Compositional data, robust variable selection, robust regression.

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Outliers and Interventions in Count Time Series

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Abstract

Time series of counts are measured in various disciplines whenever a number of events is counted during certain time periods. Examples are the monthly number of car accidents in a region, the weekly number of new cases in epidemiology, the number of transactions at a stock market per minute in finance, or the number of photon arrivals per microsecond in a biological experiment. A natural modification of the popular autoregressive moving average (ARMA) models for continuous variables is based on the assumption that the observation Y_t at time t is generated by a generalized linear model (GLM) conditionally on the past, choosing an adequate distribution for count data like the Poisson and a link function $\eta(\cdot)$. Restricting ourselves to first order models, we consider time series $(Y_t: t \in \mathbb{N}_0)$ following a Poisson model

$$\begin{aligned} Y_t | \mathcal{F}_{t-1}^Y &\sim \text{Pois}(\lambda_t), \\ \eta(\lambda_t) &= \beta_0 + \beta_1 \eta(Y_{t-1} + c) + \alpha_1 \eta(\lambda_{t-1}), \quad t \geq 1, \end{aligned} \quad (1)$$

where \mathcal{F}_{t-1}^Y stands for the σ -algebra created by $\{Y_{t-1}, \dots, Y_0, \lambda_0\}$, $\beta_0, \beta_1, \alpha_1$ are unknown parameters, and c is a known constant. The natural choice for η is the logarithm, and this is the reason for adding the constant c to Y_{t-1} in the term $\eta(Y_{t-1} + c)$ since we need to avoid difficulties arising from observations which are equal to 0. Another choice which has received some attention is the identity link, $\eta = id$, where we can set c to 0.

Given a model as formulated in (1), a basic question is whether it properly describes all the observations of a given time series, or whether some observations have been influenced by extraordinary effects, which are called interventions in what follows.

We review the intervention models for count time series proposed by Fokianos and Fried (2010, 2012) for time series which are Poisson conditionally on the past, with η being the identity and the log-link, respectively, and describe some extensions.

Key words

Generalized linear model, level shift, link function.

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Translate p-Value into Bayes Factor?

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Abstract

The p-value measures the extent to which a data refuse a hypothesis. Over the decades of its use, several fundamental flaws of the p-value were noted. For instance, the p-value is neither a coherent nor a consistent measure of refusal of the hypothesis. Some attempts were made to save the p-value by anchoring it to the Bayes factor. The Bayes factor is a bayesian measure of support for a hypothesis relative to another hypothesis. Three such a translation formulas proposed by Sellke, Bayarri and Berger [3], Good [2], Efron and Gous [1], are discussed and deficiencies of the resulting measures of support are stressed.

Key words

Evidence, p-value, Bayes factor, consistency.

References

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Semi-Copulas and Generator Triples for Fuzzy Preferences

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Abstract

We study a connection between generator triples of a fuzzy preference structure and semi-copulas. We say that a semi-copula $C: [0, 1]^2 \rightarrow [0, 1]$ is *probabilistic* if it is continuous and $C \geq T_L$.

Probabilistic semi-copulas play an important role in construction of generator triples (p, i, j) of a fuzzy preference structure, where $p(x, y)$ models strict preference of x to y , $i(x, y)$ models the indifference of x and y and $j(x, y)$ their incomparability. We assume that $p(x, y) + i(x, y) + j(x, y) + p(y, x) = 1$. The main result can be formulated as follows.

Theorem *Let C be a commutative probabilistic semi-copula and (p, i, j) be a generator triple. Further assume that $i(x, y) = C(x, y)$. Then the following statements are equivalent*

- $i(x, y) = j(1 - x, 1 - y)$,
- $p(x, y) = p(1 - y, 1 - x)$,
- $C(x, y) = \hat{C}(x, y)$, where \hat{C} is the survival semi-copula derived from C .

Key words

Fuzzy preference structure, generator triple, semi-copula.

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Detecting a Change in Annual Peak Timing

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Abstract

As discharge series usually exhibit large variability, the test for stationarity often fail to detect nonstationarities and inhomogeneities. In spite of that one may detect some changes if we do not study monthly or annual means but we rather concentrate on daily values. We started our research by studying stationarity of “annual cycles”. The annual cycle may be represented by a 365 component s vector of daily means. Splitting the series into two parts (in our case the values before the year 1997 and after it) we may compare two mean vectors of 365 components. As this problem is high-dimensional we reduced the dimensionality by the Fourier approximation method and method of principal components. The tests detected a change in mean annual cycle for 6 among 18 studied series.

The change in spring culmination timing seems to be one of the reasons why the stationarity of annual cycles has been rejected. We estimated shifts of mean and median of this timing before and after the year 1997. It is interesting to see that with one or two exceptions only these shifts are positive for all studied series. For the series where the stationarity has been rejected we calculated confidence intervals for the analyzed shifts using bootstrap method. However, the obtained intervals were broad due to a limited number of data.

Key words

Change in mean annual cycle, discharges series, statistical tests, multivariate two-sample problem, estimates of spring peak timing.

References

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Correlation Analysis for Compositional Data Using Classical and Robust Methods

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Abstract

Compositional data quantitatively describe the parts of some whole and carry exclusively relative information between the parts [1,3]. In practice, they are often represented as data with constant sum constraint (proportions, percentages) and in this form they also frequently occur in applications. Standard correlation analysis fails if it is applied directly to raw compositional data and produces in most cases complete useless results. The reason is the underlying geometry of compositional data that differs from the usual Euclidean geometry in the real space. A way out is to introduce another measures of association that can be interpreted in terms of ratios, or to express compositional data in orthonormal coordinates and to apply standard correlation analysis there. Also in real world compositional data sets, multivariate outliers frequently occur and can destroy results of the analysis in addition to the mentioned geometrical aspects. In order to suppress their influence, an affine equivariant robust estimator (like the MCD) is necessary. The theoretical considerations will be applied to a problem from official statistics.

Key words

Compositional data, isometric logratio transformation, correlation analysis, variation matrix.

References

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Simplicial Regression for Concentration-Response Experiments

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Abstract

The assessment of ecological risk from chemical contamination is of primary interest in environmental statistics. The main role in this task play dose-(or concentration)-response models that are used to compute the risk values connected to some exposure levels of a particular contaminant in living organisms. In the contribution we propose two different approaches to dose-response modeling—the standard one, where non-linear regression models are applied, and simplicial regression, based on logratio methodology for compositional data [1, 2]. The second approach follows the relative character of the proportional response and thus it seems to be more natural to use for this particular regression problem. We also apply these two approaches to real-world data [3] and compare the corresponding regression models using Akaike Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (BIC).

Key words

Dose-response modeling, non-linear regression, compositional data, simplicial regression.

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Weak Consistency—a New Approach to Consistency in the Saaty’s Analytic Hierarchy Process

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Abstract

In the decision making methods there is very important to enter the preferences of compared elements in rational way. Only in this case we can obtain reasonable solution. In the Analytic Hierarchy Process (AHP) there is set consistency condition in order to keep the rationality of preference intensities between compared elements. But this requirement for the Saaty’s matrix is not achievable in real situations because of the Saaty’s scale which is used in this method. That is why instead of consistency condition we suggest weak consistency condition which is very natural and more suitable for linguistic descriptions of Saaty’s scale and as a result of it, it is easier to reach this requirement in real situations. In addition if we order compared elements from the most preferred to the least one, it is very easy to check if the weak consistency is satisfied. Big advantage of our approach to consistency is that its satisfaction can be easily approved without using any software solution. It is also possible to control weak consistency of Saaty’s matrix during filling the intensities of preferences, not only afterwards like in the requirement set by Saaty, that consistency index CI should not be larger than 0.1. We also show that there are situations in which weak consistency condition is more suitable for checking rationality of preferences than Saaty’s consistency index.

Key words

Decision making, consistency, Saaty’s AHP.

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Replacement of Missing Values and Rounded Zeros in High-Dimensional Compositional Data with Application to Metabolomics

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Abstract

Data from metabolomics call for a specific statistical treatment. They carry only relative information, so they follow properties of compositional data [1], and usually more variables (in thousands) than samples (in tens or hundreds) occur in a data set (= high-dimensional data). Frequently, missing values and rounded zeros occur in metabolomics data sets. In particular, rounded zeros arise as values under detection limit of a measuring instrument. Although several approaches to missing values and rounded zeros imputation exist, none of them is designed to deal with high-dimensional compositional data.

The proposed algorithms for imputation of missing values and rounded zeros replacement are based on partial least squares regression that can be considered as a combination of the principal component analysis and multiple regression [2]. Theoretical developments are applied to a real-world data set from metabolomics and compared with recently used methods in the field. Also simulation results confirm that our algorithms perform well and can be used for preprocessing of metabolomics data.

Key words

Compositional data, missing values, rounded zeros, imputation.

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Comparison of the 3D Numerical Schemes for Solving Curvature Driven Level Set Equation Based on DDFV

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Abstract

In this work we describe two schemes for solving regularized level set equation in 3D based on dual finite volumes. These schemes use the so-called dual volumes as in [1], [2], where they are used for the nonlinear elliptic equation. We describe these schemes theoretically and include also compared results of the numerical experiments based on exact solution using proposed schemes.

Key words

Mean curvature flow, level set equation, numerical solution, semi-implicit scheme, discrete duality finite volume method.

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A Proper Fuzzification of Saaty's Scale and an Improvement of the Method for Computing Fuzzy Weights from Fuzzy Pair Wise Comparison Matrix

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Abstract

A proper fuzzification of the Saaty's scale for designing a multiplicative fuzzy pair wise comparison matrix and an improved method for computing fuzzy weights of elements from a multiplicative fuzzy pair wise comparison matrix will be proposed. For simplicity of explanation triangular fuzzy numbers will be used. In many fuzzifications of the AHP method that can be found in the literature, the way of the fuzzification of the Saaty's scale is not appropriate. Therefore, a new fuzzification of the Saaty's scale will be suggested. For the properly fuzzified Saaty's scale an improvement of the formulas proposed by Buckley [2] will be suggested for computing fuzzy weights from a multiplicative fuzzy pair wise comparison matrix. In the new algorithm the reciprocity condition for the pair wise comparison matrix is taken into account in each step of the computation. The fuzzy weights resulting from the proposed fuzzification of the Saaty's AHP method are less uncertain than those calculated by means of Buckley's algorithms.

Key words

Fuzzy pair wise comparison matrix, fuzzy weights, fuzzy AHP, triangular fuzzy numbers.

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Variance of Plug-in Estimators in Multivariate Regression Models

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Abstract

Variance components in regression models are usually unknown. They must be estimated and it leads to a construction of plug-in estimators of the parameters of the mean value of the observation matrix. Uncertainty of the estimators of the variance components enlarges the variances of the plug-in estimators. The aim of the paper is to find this enlargement.

The enlargement can be approximated in the form of a corection which can be obtained by a utilization of the error propagation law in its linear form. A simulation study of the statistical behaviour of the approximate estimator of the enlarged variance is presented.

Key words

Variance components, multivariate models, plug-in estimator.

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Vector Autoregression for Compositional Time Series

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Abstract

Compositional time series, CTS for short, represent time series with a possible constant sum representation at each time t (mostly the unit constant constraint is considered). This assumption constitutes the essential problem of modelling compositional time series by standard multivariate time series methods. The main approach of CTS modelling is based on using so called logratio transformations to represent the compositional time series in coordinates and apply standard statistical tools for their analysis [1]. In this contribution, an isometric logratio transformation is used for given compositional time series followed by applying a suitable VAR model [3]. It is shown that the resulting final model and predictions do not depend on the specific ilr transformation used. In addition, particular care was devoted to the proper choice of balances [2] which makes the corresponding statistical inference like hypothesis testing easily interpretable. The application to real data demonstrates that the standard non-compositional approach can result in misleading predictions.

Key words

Compositional time series, VAR model, isometric logratio transformation, simplex.

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A Continuation Problem for Computing Solutions of Discretised Quasi-Static Problems

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Abstract

Let us consider incremental problems of the form

$$\mathbf{G}\left(\mathbf{x}^{k+1}, \frac{\mathbf{x}^{k+1} - \mathbf{x}^k}{t_{k+1} - t_k}\right) = \mathbf{F}(t_{k+1}, \mathbf{x}^{k+1}) \quad (\mathcal{P}_{k+1})$$

that correspond to a discretised nonlinear quasi-static problem, possibly involving some viscous terms. Here, $\mathbf{x}^k, \mathbf{x}^{k+1} \in \mathbb{R}^N$, $\mathbf{G}: \mathbb{R}^N \times \mathbb{R}^N \rightarrow \mathbb{R}^N$ and $\mathbf{F}: \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}^N$. When solving such problems numerically, one can encounter situations where usual solvers (for example, the Newton method with the initial approximation \mathbf{x}^k) fail to compute a solution and more sophisticated techniques have to be employed.

To start with, we introduce a suitable continuation problem that can be written as

$$\mathbf{H}_{k+1}(\gamma, \mathbf{x}) = \mathbf{0} \quad (2)$$

with $\mathbf{H}_{k+1}: \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}^N$ and $\gamma \in \mathbb{R}$ being a continuation parameter. The function \mathbf{H}_{k+1} is constructed so that any \mathbf{x}^k solving the problem (\mathcal{P}_k) satisfies $\mathbf{H}_{k+1}(0, \mathbf{x}^k) = \mathbf{0}$ and \mathbf{x} solves (\mathcal{P}_{k+1}) if and only if $\mathbf{H}_{k+1}(1, \mathbf{x}) = \mathbf{0}$. Hence, the couple $(0, \mathbf{x}^k)$ can always be chosen as an initial point for numerical continuation of (2) and any couple (γ, \mathbf{x}) with $\gamma = 1$ found during the continuation yields a solution of the problem (\mathcal{P}_{k+1}) we seek primarily.

Next, we deal with continuation of (2). Whereas this issue is well explored under the assumption of continuous differentiability of \mathbf{H}_{k+1} , we suppose the function to be only *piecewise* differentiable. In this framework, the first-order system of (2) is derived and analysed. Possibility of continuation of solution paths along one-sided tangent directions coming from the first-order system is studied. Finally, the continuation algorithm described in [1] is adopted for following solution branches numerically.

Key words

Continuation, piecewise differentiable function, first-order system.

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Solution of Contact Problems for a Beam and an Elastic Foundation

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Abstract

This report follows the articles [1] and [2] where a bending of a beam resting on an elastic foundation was considered. These works were aimed especially on the case with the so-called unilateral foundation, which represents some nonlinear contact problem. The solution procedure consists in a problem decomposition followed by introducing a Lagrangian and transferring the original problem to a saddle-point problem.

Now we want to consider also beams positioned above a foundation or more generally – above an elastic obstacle. By means a similar decomposition we can establish a solution procedure that is working also for this case.

In addition, we are able to solve such contact problems using the idea from the paper [3] as well. The control variational method is a modification of the classical variational approaching, via the use of optimal control theory. It is worthwhile both from the theoretical and the numerical points of view.

Finally we are going to present some algorithms based on the finite element discretization and the above mentioned solution methods.

Key words

Euler–Bernoulli beam, contact problems, decomposition method, control variational method.

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Canonical Duality in Optimization Problems

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Abstract

Duality in optimization is essentially a problem transformation that can lead to sometimes more efficient solution method. The most used is the Lagrange's concept of duality. The original problem, which is referred to as the primal problem, is transformed into a problem in which the parameters are the multipliers of the primal. The transformed problem is called the dual problem. The duality gap measures the difference between the dual and primal optimal values. When there is no duality gap, primal problem can be solved by means of its dual, if it is more convenient. This works good for convex problems. But there are a lot of nonconvex optimization problems as well and in such cases the Lagrangian duality creates nonzero duality gaps. Hence this procedure is for nonconvex cases in principle useless.

Many researchers wanted to overcome this problem with more or less success. Very remarkable results achieved Prof. David Yang Gao, who took his inspiration from mechanics problems. In his so-called canonical duality transformation play fundamental roles the Legendre transformation and the concept of canonical functions. This approach resulted into readjustment of the most parts of the Lagrange's theory and led to the so-called bi-duality and triality theory with connections to global optimization (see [1], [2]).

We want to present here some interesting applications of the canonical duality theory. Especially we are going to consider the general nonconvex quadratic optimization and a very important practical problem from mechanics which is concerned with the nonlinear beam model discovered by D. Y. Gao in 1996.

Key words

Classical Lagrangian duality, nonconvex optimization problems, canonical dual transformations, Gao beam model.

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Discrete Averaged Mixed Distributions

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Abstract

The usual mixing of distributions is performed by weighting the individual components in such a way that the sum of positive weights yields 1. There is also a second type of mixing, namely the randomization of one or more parameters of the basic distribution by another distribution. The large classes of these two types are sufficiently known (see [1], Chapter 8 or [2]). We present a new type of mixing—addition (or subtraction) of the non-normalized components of “standard” distributions divided by the sum (or difference) of the normalizing constants. This class of distributions will be called discrete averaged mixed distributions. Here we present a subclass of such distributions, namely averaged mixed logarithmic distributions: their general form, basic probabilistic characteristics such as probability generating function, probability mass function, moments, genesis of the distributions and their special cases. In general they have the probability generating function of the type

$$G(t) = \frac{\log \frac{1-bt}{1-at}}{\log \frac{1-b}{1-a}},$$

a and b being parameters.

Key words

Discrete distributions, mixed distributions, weighted distributions.

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Lateral Asymmetry and Torsional Oscillations in Suspension Bridges

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Abstract

The original Tacoma suspension bridge was completed on 10 June 1940 and opened to traffic on 1 July 1940. The bridge was stable with respect to torsional oscillation until 7 November 1940. That day at 10 a.m. the diagonal tie attached to the midspan band of one main cable loosened and the cable began to slip through the band. Just after the loosening of the tie torsional oscillations appeared, lasted for more than one hour, and resulted in the collapse of the center span at 11:10 a.m. In this paper a continuous model of the original Tacoma suspension bridge is proposed. This model describes the mutual interaction of the main cables, central span, and cable stays. The reaction of the ties attached to the midspan bands is included in the model, so it is possible to study the situation when only one midspan band loosens. The model is described by a system of variational equations which are derived from the Hamilton variational principle. Three different eigenvalue and eigenvector problems are formulated and analyzed. The problems correspond to the situations when the both midspans are loosened, the both midspan bands are fixed, and one midspan band is fixed and the other is loosened. The analysis of the three eigenvalue and eigenvector problems against flutter is carried out, which reveals possible reasons of the collapse (see [1]).

Key words

Lateral asymmetry and torsional oscillations, suspension bridges.

References

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Principal Balances with Sparse PCA

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Abstract

Principal component analysis (PCA) transforms the variables of a dataset into new variables, called principal components (PCs) which do not correlate with each other and subsequently maximize the explained variance. It is thus frequently used for dimension reduction. Applying PCA to clr-transformed compositional data results in loadings that form an orthonormal basis of the simplex, and in scores that are ilr coordinates. The ilr coordinates are usually difficult to interpret since they involve all compositional parts to some extent. This has been solved by the concept of balances [1], orthonormal coordinates consisting of two non-overlapping groups of compositional parts. Balances can be constructed by a sequential binary partition [1], but this construction does not consider a criterion like maximizing the explained variance.

For this reason, [2] defined principal balances (PBs) which are balances that are subsequently constructed to maximize the explained (remaining) variance. The computation of PBs requires an exhaustive search among all possible sets of orthogonal balances, which is computationally infeasible especially for high-dimensional data. [2] thus introduced different approximative algorithms using the PCs of the clr-transformed data, and based on hierarchical clustering. In an example with 21 compositional parts they have demonstrated the usefulness of this approach.

In this paper we want to go further. We propose an algorithm to compute PBs for data sets with hundreds or even thousands of parts, as it is typical in chemometrics (e.g. mass spectral data) or biology (e.g. gene expression data). The basic assumption is that only a small subset of parts contributes to the main data variability. This requirement is typically met with microarray data, where only few genes are responsible for group separation, causing the essential data variability.

The proposed algorithm is based on sparse PCA (SPCA) [3] applied to the clr-transformed data. SPCA finds components that maximize the variance, with an additional penalty term to take sparseness into account. The result is a sparse loading matrix containing many zeros, which makes the interpretation easier. The penalty parameter, say λ , regulates the priority between variance maximization and sparseness, and it can be determined by a BIC-type criterion [3]. The resulting loadings of the first k sparse PCs still need to be converted to balances. We will call them *sparse principal balances*. We suggest a sequential procedure: The signs of the first sparse PC loadings are used in the usual way to form the first balance. Since for a balance we always need at least one positive and one negative sign, the penalty parameter λ is optimized not only according to the BIC, but also to include this condition. From subsequent sparse PCs we only use those signs for defining a balance which were not used for previously.

It turns out that for high-dimensional data this approach leads to a more efficient dimensions reduction—in terms of explained variance—than the algorithms of [2]. Our method is illustrated with simulated and real high-dimensional data.

Key words

Principal component analysis, compositional data, isometric log ratio transformation.

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Integrals: Special Functionals and an Optimization Tool

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Abstract

Considering the space of all non-negative measurable functions $\mathcal{F}_{(X,\mathcal{A})}$ linked to a fixed measurable space (X, \mathcal{A}) , the Lebesgue integral can be seen as an additive, continuous from below functional \mathcal{L} on $\mathcal{F}_{(X,\mathcal{A})}$, related to a measure $m: \mathcal{A} \rightarrow [0, \infty]$ given by $m(A) = \mathcal{L}(1_A)$. We introduce several other integrals which can be seen as special functionals on $\mathcal{F}_{(X,\mathcal{A})}$. For example, the Choquet integral is a comonotone additive functional \mathcal{C} , and the corresponding monotone measure $m: \mathcal{A} \rightarrow [0, \infty]$ is given by $m(A) = \mathcal{C}(1_A)$. Similarly, the Sugeno integral $\mathcal{S}u$ is a min-homogeneous comonotone maxitive functional on $\mathcal{F}_{(X,\mathcal{A})}$. Some integrals were introduced as special functionals. We recall the convex integral $\mathcal{C}l$ introduced by Lehrer [2], which, for a given monotone measure m on \mathcal{A} , is the smallest positively homogeneous concave functional on $\mathcal{F}_{(X,\mathcal{A})}$ such that $\mathcal{C}l(1_A) \geq m(A)$.

On the other hand, several new types of integrals were or can be introduced as optimization tools. For a given system \mathcal{H} of set systems from \mathcal{A} , Even and Lehrer [1] have proposed the decomposition integral $\mathcal{D}_{\mathcal{H}}: \mathcal{F}_{(X,\mathcal{A})} \rightarrow [0, \infty]$, which, for a given monotone measure m on \mathcal{A} , is given by

$$\mathcal{D}_{\mathcal{H}}(f) = \sup \left\{ \sum_{i \in I} a_i m(A_i) \mid a_i \geq 0, (A_i)_{i \in I} \in \mathcal{H}, \sum_{i \in I} a_i 1_{A_i} \leq f \right\}.$$

Obviously, if \mathcal{H} consists of finite (countable) partitions of (X, \mathcal{A}) and m is a measure, then $\mathcal{D}_{\mathcal{H}} = \mathcal{L}$. If \mathcal{H} consists of all finite (countable) chains, then $\mathcal{D}_{\mathcal{H}} = \mathcal{C}$. If \mathcal{H} consists of all singleton systems, $\mathcal{H} = \{\{A\} \mid A \in \mathcal{A}\}$, $\mathcal{D}_{\mathcal{H}} = Sh$ is the Shilkret integral. Several other new types of decomposition integrals will be shown, too, see [3].

Key words

Decomposition integral, universal integral, multicriteria decision support, optimization.

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Hygic Models in Enclosure

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Abstract

Moisture problems are still persisting in buildings. Indoor climate of the rooms where people reside should fulfill standard requirements stipulated by norms. Among them, the indoor air temperature and relative humidity have to fall within the specified range. In the same time the surfaces of the walls, windows and doors have to be of the temperature above the temperature ensuring the mould and condensation alleviation.

The hygic model of a room evaluates the partial pressure p_i of the water vapor in the air. The input parameters are outdoor air temperature T_e [K], indoor air temperature T_i [K], water vapor production G_p in the room, volume V [m³] of the room, outdoor humidity represented by partial pressure p_e [Pa], ventilation intensity n [s⁻¹], and suction capability of the surfaces.

Neglecting the suction of the surrounding surfaces (simplified model), the mass balance is $G_p + G_i = G_o + G_s$. Here G_p [kg/s] quantifies the vapor per time produced in the room, G_i the amount of incoming vapor by the ventilation, G_o the amount of outgoing vapor and G_s vapor stored in the room. By using the ideal gas equation $p \cdot V = m \cdot R \cdot T$ with R [J/(kg·K)] the gas constant we can write

$$G_i = \frac{nVp_e}{RT_i}, \quad G_o = \frac{nVp_i}{RT_i}, \quad G_s = \frac{V}{RT_i} \frac{dp_i}{dt_i}.$$

Vapor uptake by hygroscopic surfaces $\sum_k \bar{G}_k$ acts as a dumping factor. The mass balance is

$$G_p + G_i = G_o + G_s + \sum_k \bar{G}_k.$$

After rearranging we get the extended—more realistic model

$$\frac{dp_i}{dt_i} = n(p_e - p_i) + \frac{RT_i (G_p + \sum_k \bar{G}_k)}{V}.$$

Key words

Heat equation, saturation line, hybrid model, moisture, building engineering suction isotherms, FEM modeling.

References

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An Application of Survival Analysis to Dataset of Patients with Hodgkin Lymphoma

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Abstract

Survival analysis is one of the methods which can deal with the longitudinal time-to-event studies. Most of these studies come from the medical field. The data used for survival analysis are different from other statistical data because of long flow-up time and especially because of the censoring. The useful statistical method for analysing these type of data is the Kaplan–Meier estimator of survival curve and the Cox proportional regression model. The both methods were used for analysing patients suffered by Hodgkin lymphoma. The study contains 194 patients who were divided into three groups. There were reduced a relative dose intensity (RDI) of chemotherapy in the two groups and 100 % dose intensity (RDI) of chemotherapy in the last group. The aim of the study was to show that reducing of RDI has an impact on surviving of the patients. The Kaplan–Meier curves were used for representation of overall survival and event-free survival of patients. The Cox models were used for analysing complete remission and disease relapse.

Key words

Survival analysis, Kaplan–Meier estimator, Cox model, Log Rank test.

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Vehicle Routing, Data Uncertainty and Metaheuristic Algorithms

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Abstract

The Vehicle Routing Problem (VRP) is a combinatorial optimization problem arising in logistic applications. In this talk, after having formally defined the problem, we will consider a variant of the VRP where input data (travel times, service times at customers, etc.) are affected by uncertainty, as it likely happens in the reality. Concept from Robust Optimizations will be introduced and it will be shown how they can be efficiently embedded into an Ant Colony Optimization heuristic algorithm. The resulting approach is shown to produce robust solutions that are not only optimized, but also protected against uncertainty.

Key words

Vehicle routing, uncertain data, ant colony optimization.

Relative Arithmetic for Non-Compatible Observables

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Abstract

The aim of this contribution is to show how it is possible to introduce a relative arithmetic for non-compatible observables. The importance of the existence of sum of observables modelling non-compatible events was emphasized already by J. von Neumann in [1]. Many researchers tried to solve this problem [2].

Our approach is based on conditional states (conditional normalized measures) [3] on an orthomodular lattice. This problem is equivalent with the problem of existence of bivariate state [4].

Key words

Conditional state, non-compatibility, orthomodular lattice, homomorphism.

References

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Minimum Distance Estimators in Measurement Error Models

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Abstract

Measurement error models (also called errors-in-variables models) are regression models that account for measurement errors in the independent variables (regressors). These models occur very commonly in practical data analysis, where some variables cannot be observed exactly, usually due to instrument or sampling error. Sometimes ignoring measurement error may lead to correct conclusions, however in some situations it may have dramatic consequences.

History of these models is very rich, it started at the end of the nineteenth century. Since then various methods for dealing with measurement errors were developed, such as least squares estimates, maximum likelihood estimates and most recently total least squares estimates. Anyway, nonparametric methods are not very common, although they suit to this situation well due to the absence of knowledge of the measurement error's distribution.

We will introduce a class of minimum distance estimators into linear regression model with stochastic regressors that may be subject to measurement error. To do it we first consider estimates in model where regressors and model errors are not independent, but only uncorrelated. This result will be then extended into measurement error models. As a byproduct we also introduce a class of distribution-free rank tests for testing in measurement error models.

All the theoretical results will be illustrated on examples and simulations. Suggested estimates and tests will be compared with standard ones and will be shown their good performance, for some situation ever better than classical approaches.

Key words

Minimum distance estimates, measurement errors, rank tests.

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Intermediate Quantifiers in Fuzzy Natural Logic

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Abstract

We will present a formal theory of intermediate quantifiers that are linguistic expressions such as “most”, “many”, “few”, “almost all”, etc. This theory was initiated by V. Novák in [3] and continued in [1]. We were inspired by the book of P. L. Peterson [5]. The main idea consists in the assumption that intermediate quantifiers are just classical quantifiers \forall or \exists whose universe of quantification is modified using an evaluative linguistic expression. The latter is an expression such as “very small, extremely large, roughly big, more or less medium”. Because the meaning of evaluative expressions is imprecise, the meaning of intermediate quantifiers is imprecise as well. The formalization is realized as a formal special theory of higher-order fuzzy logic, namely Łukasiewicz fuzzy type theory. Recall that the fuzzy type theory was introduced by V. Novák in [2]. We can demonstrate that over 120 generalized syllogisms with intermediate quantifiers are valid in our theory. We will also provide analysis of the *generalized Aristotelian square* of opposition that extends the classical one by several selected intermediate quantifiers. Let us emphasize that most proofs proceed syntactically. This makes our theory very general and strong since we can find various kinds of models in which the proved properties hold. Among other outcomes, this makes our theory also very practical with high potential for real applications. Such applications are in preparation.

Key words

Mathematical fuzzy logic, fuzzy type theory, generalized quantifiers, intermediate quantifiers, generalized Aristotle syllogisms.

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Proportional Representations of Interval Compositional Data

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Abstract

In statistics, compositions data are defined as multivariate observations that quantitatively describe contributions of parts on a whole, carrying exclusively relative information [1]. Geometric properties of compositional data with the simplex as their sample space are characterized by the Aitchison geometry [2]. As a consequence, such data can be represented as proportions or percentages without loss of information (that is contained in ratios between parts). Nevertheless, in practice frequently such situations occur, when parts of compositions data are represented as intervals. E.g. concentrations of chemical elements are provided not as exact numbers, but rather in an interval range. Consequently, we are interested, whether also in such a case the relative information (i.e. ratios) is preserved, when the original composition data with interval values are represented in proportions. Namely, from arithmetic of interval data, normalizing of intervals does not simply follow the case of real values, but a special procedure according to constrained interval arithmetic is needed [3].

The aim of the contribution is to discuss possibilities of representing the interval compositional data in proportions with respect to geometric mean as a measure of central tendency of compositions with respect to the Aitchison geometry [4].

Key words

Compositional data, Aitchison geometry, geometric mean, constrained interval arithmetic.

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Thanks

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White-Light Interferometry—Coherence Peak Evaluation and Influence of Noise

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Abstract

White-light interferometry is an established and proved method for the measurement of the geometrical shape of objects [1]. An important advantage of white-light interferometry is that it can measure the shape of objects with smooth as well as rough surface. The information about the longitudinal coordinate of the surface of the measured object is obtained from the so called white-light interferogram. The object to be measured is placed in one arm of a Michelson interferometer and moved in the longitudinal direction during the measurement procedure. The interferogram is the intensity at the output of the interferometer expressed as the function of the position of the object. A CCD camera is usually used as the detector at the output of the Michelson interferometer. If the surface of the measured object is rough, the image at the output of the interferometer is speckled. Because of the influence of the speckle field, the phase of the interferogram is a random value. The information about the longitudinal coordinate of the respective surface point is obtained from the position of the coherence peak. The coherence peak is represented by the envelope of the interferogram. A classical method for the calculation of the envelope of white-light interferogram is the demodulation by means of Hilbert transform. However, the signal at the output of the CCD camera is influenced by the noise. Therefore, as expected, the calculated envelope is also influenced by the noise. The measurement uncertainty is calculated by means of Cramér–Rao inequality. However, in the train of the demodulation, the noise of the envelope becomes correlated. Therefore, when the Cramér–Rao inequality is applied, the inverse of the correlation matrix must be taken into account. In our research, we look for the answer on following questions: How does the noise of the evaluated envelope differ from the noise of the interferogram? What is the minimal measurement uncertainty that can be achieved?

Key words

White-light interferometry, Hilbert transform, noise, Toeplitz matrix, measurement uncertainty.

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The product space \mathbf{T}

(tools for compositional data with a total)

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Abstract

Compositional data analysis usually starts with data presented as vectors in the positive orthant of D -dim. real space, \mathcal{R}_+^D . If interest lies solely in comparing data independently of their size, standard practice is to project them in the simplex \mathcal{S}^D , the subset of \mathcal{R}_+^D that results from constraining the data to sum to a constant κ . The projection is attained applying the *closure operation* \mathcal{C} [1], which consists in dividing each component by the sum of all components.

This approach enables compositional analysis, but ignores information about the total (abundances, mass, amount, ...). To analyse the whole vector, two alternatives are usual in practice. One consists in considering $\ln(\mathbf{x})$, where the logarithm applies componentwise. The other one involves projecting the composition into \mathcal{S}^D and considering the total sum as an additional variable. Although similar, these alternatives are not equivalent and they differ in the treatment of the total amount considered. To perform an analysis of the obtained vectors using standard methods, it is necessary to go a step further and express them as coordinates in real space [2]. We examine, following [3], which are suitable Euclidean structures of the support spaces, with the assumption that components are strictly positive. Zeros require a special treatment, similar to that used for compositional data. Real data about total abundances of phytoplankton in an Australian river are used for illustration.

Key words

Aitchison geometry, compositional data, simplex.

References

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Thanks

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Computational Intelligence for Image Processing

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Abstract

There are many methods involved in image processing. A vast majority of them consider an image as a two-dimensional signal, and process it accordingly. As a result, the technique of Fourier transform that is widely used in signal processing is also used in the image processing. It requires to formulate a specific problem in the language of frequencies and solve it in the corresponding space. However, in many problems that are specific for the image processing, this approach leads to the situation where the connection between a problem and a method of its solution is lost.

Contrary to the above, we propose to consider and process a 2D image taking it as a luminance function of two variables defined on a respective set of pixels. This allows us to stay in the same space where a problem is formulated and solved. We consider some typical problems of image processing: compression, reconstruction, edge detection, and explain how these problems can be formulated in the language of functions and their characteristics. Moreover, we propose a unique technique that can be used for successful solution of all the chosen problems and explain why and how it works. This technique is called fuzzy (F-)transform and is based on a fuzzy partition of a universe of discourse.

The theory of F-transform is a modern theoretical tool for fuzzy modeling. On the basis of this theory, a methodology with many applications in areas of data analysis, image processing, time series analysis and forecasting has been developed. Due to clear theoretical foundations and common effort of many researches, both theory and applications have been extensively developed in recent years.

In the talk we plan the following:

- to introduce backgrounds of the F-transform methodology including simple and attractive properties of its direct and inverse parts,
- to show and explain successful applications of the F-transform methodology in image processing and computer vision.

Key words

Image processing, fuzzy transform, compression, edge detection.

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Thanks

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Steady Compressible Navier–Stokes–Fourier System with Temperature Dependent Viscosities

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Abstract

We consider the system of partial differential equations in $\Omega \subset \mathbb{R}^3$

$$\operatorname{div}(\varrho \mathbf{u}) = 0, \quad (3)$$

$$\operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) - \operatorname{div} \mathbb{S} + \nabla p = \varrho \mathbf{f}, \quad (4)$$

$$\operatorname{div}(\varrho E \mathbf{u}) = \varrho \mathbf{f} \cdot \mathbf{u} - \operatorname{div}(p \mathbf{u}) + \operatorname{div}(\mathbb{S} \mathbf{u}) - \operatorname{div} \mathbf{q} \quad (5)$$

which describes the steady flow of a heat conducting compressible fluid in a bounded domain Ω . We consider (3)–(5) together with the boundary conditions at $\partial\Omega$

$$\begin{aligned} \mathbf{u} \cdot \mathbf{n} &= 0, \\ (\mathbb{S} + \lambda \mathbf{u}) \times \mathbf{n} &= \mathbf{0}, \end{aligned} \quad (6)$$

$$-\mathbf{q} \cdot \mathbf{n} + L(\vartheta - \Theta_0) = 0. \quad (7)$$

We assume the fluid to be Newtonian, i.e.

$$\mathbb{S} = \mathbb{S}(\vartheta, \nabla \mathbf{u}) = \mu(\vartheta)(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3} \operatorname{div} \mathbf{u} \mathbb{I}) + \xi(\vartheta) \operatorname{div} \mathbf{u} \mathbb{I},$$

with the pressure $p(\varrho, \vartheta) \sim \varrho \vartheta + \varrho^\gamma$ and the heat flux $\mathbf{q}(\vartheta, \nabla \vartheta) = -\kappa(\vartheta) \nabla \vartheta$. We study existence of a solution to our problem (3)–(7) in dependence on γ , α and m , where $\mu(\vartheta)$, $\xi(\vartheta) \sim (1 + \vartheta)^\alpha$, $\kappa(\vartheta) \sim (1 + \vartheta)^m$. We prove existence of variational entropy solutions for any $\gamma > 1$ and suitable α and m , which is in some cases even a weak solution to our problem. The talk is based on recent papers [1] and [2].

Key words

Compressible Navier–Stokes–Fourier system, steady solution, weak solution, variational entropy solution.

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Thanks

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State-morphism Pseudo-effect Algebras

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Abstract

The notion of a state-morphism on pseudo-effect algebras is introduced, and pseudo-effect algebras with distinguished state-morphisms are studied under the name state-morphism pseudo-effect algebras (SMPEAs). It is shown that every SMPEA admits a representation as a (total) state-morphism algebra, and some results from the general theory of state-morphism algebras (that is, algebras endowed with a distinguished idempotent endomorphism called a state-morphism), recently developed by Botur and Dvurečenskij, can be applied. In particular, it is shown that under suitable conditions, a SMPEA can be embedded into a so-called diagonal one, realized by a direct product of the SMPEA with itself endowed with a suitable natural state-morphism.

Key words

State operator, state-morphism, pseudo effect algebra, Riesz congruence, normal Riesz ideal.

References

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Thanks

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On the Kluvánek Construction of the Lebesgue Integral

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Abstract

I. Kluvánek suggested to built the Lebesgue integral on a compact interval in the real line by the help of the length of intervals only. In the paper a modification of the Kluvánek is presented applicable to abstract spaces, too.

Let \mathcal{J} be the family of all subintervals of a given interval $[a, b]$. If $A \in \mathcal{J}$, then $\lambda(A)$ is the length of A , i.e.

$$\lambda([c, d]) = \lambda([c, d]) = \lambda((c, d]) = \lambda((c, d)) = d - c.$$

In the following definition the Kluvánek construction is presented.

Definition $f \in \mathcal{K} \iff$

$$\begin{aligned} \exists \alpha_i \in \mathbb{R} \exists A_i \in \mathcal{J}, \sum_{i=1}^{\infty} |\alpha_i| \lambda(A_i) < \infty \\ \sum |\alpha_i| \chi_{A_i}(x) < \infty \implies f(x) = \sum \alpha_i \chi_{A_i}(x) \end{aligned}$$

The only didactical problem is in the proof of the independence of the integral of a function f on the presentation in the form

$$f(x) = \sum \alpha_i \chi_{A_i}(x)$$

Therefore we suggest a modification of the construction considering first non-negative functions only. In the first part of our paper the equality of two definitions is shown. In the second part it is shown the independence of the sum

$$\sum_{i=1}^{\infty} \alpha_i \lambda(A_i)$$

($\alpha_i \geq 0$) on the representation of f in the form

$$f(x) = \sum \alpha_i \chi_{A_i}(x).$$

Key words

Lebesgue integral, Kluvánek construction of the Lebesgue integral.

References

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Statistical Inference about the Drift Parameter in Stochastic Processes

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Abstract

In statistical inference on the drift parameter a in the Wiener process with a constant drift $Y_t = at + W_t$ there is a large number of options how to do it. We may, for example, base this inference on the properties of the standard normal distribution applied to the differences between the observed values of the process at discrete times. Although this method is very simple and it is easy to implement, it turns out that more appropriate is to use the sequential methods. In the presentation the inference is based on the first hitting time of a given open interval, mostly until some given time. These methods can be implemented and are numerically compared and illustrated. For the hypotheses testing about the drift parameter it is more proper to standardize the observed process and to use the sequential method based on the first time when this process reaches B or $-B$, where $B > 0$, until some given time. the drift parameter under non-constant drift assumption. Because of the symmetry and the martingale property of the Wiener process, we can generalize such a method if we consider the Itô integral of a known deterministic function b , i.e. $(\int_0^t b(s)dW_s, t \geq 0)$, instead of the ordinary Wiener process $(W_t, t \geq 0)$. Another generalizations can be done for the Brownian bridge, etc.

Key words

Wiener process, Itô integral, martingale, sequential methods.

References

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Thanks

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Peer-review Integration into the R&D Evaluation Based on Objective Data—a Model Using Fuzzy AHP

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Abstract

In this paper we aim to contribute to the current discussion concerning the proper approach to the evaluation of R&D outcomes in the Czech Republic. In consistency with the recommendation of [1] we introduce a mathematical model that combines an expert evaluation of publisher with an expert evaluation of the particular R&D outcome. The presented model is currently being used for the evaluation of books and book chapters at the Faculty of Science, Palacký University in Olomouc [2]. The outputs of the evaluation based on the presented model are used for the distribution of funds among departments. Four categories of publishers are defined based on the reputation of the publisher (taken as an objective criterion). These categories are then compared pair-wise by a group of experts, who express their preferences (reflecting the goals of the Faculty) between these categories using the Saaty's matrix of preference intensities [3, 4]. The weights of categories are then computed using the fuzzified AHP introduced [5]. This way the uncertainty of linguistic expressions represented by elements of the properly fuzzified Saaty's scale [5], which are used to describe the intensities of preferences between categories can be properly reflected in the computation. The result is a triangular fuzzy number representing possible evaluations of books published by a publisher from the respective reputation category. This fuzzy number is then interpreted as an interval of scores (the support) with a most typical score (default evaluation—the only element in the kernel of the fuzzy number). In the following peer-review process (second stage) the default evaluations can be altered within the limits of proposed interval of scores for the respective category. Comparison of results of the classical AHP with the fuzzified AHP will be presented.

Key words

Linguistic fuzzy modeling, decision making support, linguistic scale, emergency medical rescue services.

References

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Discretization and numerical methods for the Hencky elastic-perfectly plastic problem

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Abstract

The contribution summarizes some of the results introduced in [1] and [2]. In comparison with these references, only the classical boundary conditions are considered for the sake of brevity. The investigated elastic-perfectly plastic model is given by the Hencky law with the von Mises yield criterion.

The problem has been studied in many papers summarized in [1] and [2], where its theoretical and related numerical difficulties are described. The problem has a unique solution in terms of stresses provided that there exists a statically and plastically admissible stress field. The existence of such a field is dependent on the load, represented by the functional L . Therefore the load is usually multiplied by a parameter $\lambda \geq 0$, where λ is enlarged up to the maximal admissible (limit) value $\bar{\lambda} > 0$ ensuring the solvability. In terms of displacements, the corresponding minimization functional is not coercive in general on reflexive Sobolev spaces and non-separable BD-spaces must be used for solvability analysis. Therefore, a suitable discretization of the problem is still an open problem.

The contribution is focused on a standard (convex) finite element discretization and its evaluation. Since the discretized problem in terms of displacement is not fully compatible with the continuous problem, analysis of the finite-dimensional problem is studied in detail. In particular, it is investigated the following: (i) dependence of a solution set to the problem on the load parameter λ , (ii) dependence of λ on a parameter α representing the work of external forces. It enables to characterize and evaluate the loading process.

In a numerical part of the contribution, we consider Newton-like and augmented Lagrangian methods. Their convergence analysis is studied in terms of stresses and displacements. Some numerical experiments are performed.

Key words

Elasto-perfect plasticity, FEM discretization, limit analysis.

References

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Fuzzy Comparison Measures and Fuzzy OWA in Multicriteria Decision Making

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Abstract

This paper proposes a multicriteria decision making (MCDM) method where fuzzy scorecards are used for initial collection of evaluations for fuzzy decision matrixes for each alternative and criterion. Decision matrixes are aggregated over the criterias by using fuzzy OWA operator [1], [2]. After this, these aggregated fuzzy numbers are compared to so called ideal solution. To make the comparison and form rankings between the fuzzy numbers formed this way is not a trivial task since fuzzy numbers bears a lot more information with them than the crisp ones. Besides this also subject of how to select the appropriate ideal solution clearly has major effect in this process. Here we concentrate on review and test what type of comparison measures exists on finding a total ordering of the alternatives gained from forementioned comparison. Also the subject of how to form fuzzy ideal solution is examined. For the comparison measures we have examined all together 12 different comparison measures for fuzzy numbers. These include measures based on cardinality, fuzzy similarity measures, fuzzy distances and correlation indexes. Besides this also two stage measures are examined, which does not fall under previous category, but are build so that several different properties are included in the measure, i.e. cardinality, entropy, moment, skewness etc. From the fuzzy similarity measures we have examined measures, which are able to consider several different types of information. These include i.e. geometric distances, perimeters and areas of fuzzy numbers [3].

Key words

Fuzzy decision making, fuzzy OWA, fuzzy comparison measures.

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Outlier detection in compositional data with structural zeros

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Abstract

Many compositional data sets include structural zeros (values that are truly observed and being zero), sometimes with concentration of more than 50 % of the values. Examples of structural zeros are plant species that are not able to survive in a given soil type or climate, a political party that has no candidates in a region, or teetotal households that do not have expenditures on alcohol and tobacco. The presence of structural zeros causes serious difficulties for the analysis and standard transformations, and methods from a log-ratio approach to compositional data analysis can not be used.

In this contribution we focus on outlier detection for compositional data and enhance the work from [1] and [2] by allowing structural zeros in the data. We also employ the Mahalanobis distance approach for outlier detection, and thus one goal is the robust estimation of the covariance matrix.

The first method is based on imputation of zeros, transformation (using the so called isometric log-ratio transformation) and covariance estimation on non-zero parts. The second method proposed is based on the variation matrix, which is estimated pairwise and robustly from non-zero (bivariate) parts.

In contrast to existing methodology, the imputation based method shows an excellent performance in real-world applications and simulation studies that have been carried out.

Our approach is not only suitable for outlier detection, it can be used for all covariance based methods like principal component analysis or discriminant analysis and is not only restricted to structural zeros—it can be applied also for data with missing values.

Key words

Compositional data, structural zeros, outlier detection.

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Robust R Tools for Exploring High-Dimensional Data

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Abstract

High-dimensional data are typical in many contemporary applications in scientific areas like genetics, spectral analysis, data mining, image processing, etc. and introduce new challenges to the traditional analytical methods. First of all the computational effort for the anyway computationally intensive robust algorithms increases with increasing n and p towards the limits of feasibility. Some of the robust multivariate methods available in R (see [3]) are known to deteriorate rapidly when the dimensionality of data increases and others are not applicable at all when the number of variables p is larger than the number of observations n . The present work discusses robust multivariate methods specifically designed for high dimensions. Their implementation in R is presented and their application is illustrated on a number of examples. A key feature of this extension is that object model follows the one already introduced by **rrcov** and based on statistical design patterns. The first group of classes are algorithms for outlier detection, already introduced elsewhere and implemented in other packages. The value added of the new package is that all methods follow the same pattern and thus can use the same graphical and diagnostic tools. The next topic covered is sparse principal components including an object oriented interface to the standard method proposed by Zou et al. [4] and the robust one proposed by Croux et al. [1]. Robust partial least squares (Hubert and Vandenberg, [2]) as well as partial least squares for discriminant analysis conclude the scope of the new package. All considered methods and data sets are available in the R package **rrcovHD**.

Key words

Robustness, Multivariate, Outliers, PCA, PLS, rrcov.

References

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Depth Based Classification: Several New Ideas

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Abstract

The present contribution deals with ongoing research of data depth and its applications. It recalls the notion of data depth—a relatively new nonparametric approach to analysis of multivariate data. The data depth concept and its possible applications was discussed in detail at the ODAM 2011 conference followed by the paper [1] which provides a review of possible applications of the data depth concept.

The present contribution is focused on depth based methods of classification. These methods have been developed in the last ten years. Recently, several modifications of classical nonparametric procedure called k -nearest-neighbours was proposed. We show benefits of these modifications and compare them in a short simulation study.

Key words

Data depth, classification, kNN, modification, simulation.

References

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Predictive Distributions in Fuzzy Bayesian Inference

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Abstract

The concept of predictive distributions has to be generalized to the situation of fuzzy data and fuzzy a-priori distributions. This was proposed in different ways. Recently a novel approach has been developed based on families of classical probability densities on the parameter space. Details will be given in the contribution.

Key words

Fuzzy numbers, Fuzzy Bayesian inference, Predictive distributions.

References

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Negotiation, Bargaining, Arbitration

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Abstract

Since John Nash's papers on two-person bargaining [2], [3] and Howard Raiffa's studies on arbitration [5], [6] in the beginning of 1950's, it has become customary to present the data defining an n -person bargaining problem as a nonempty collection \mathcal{B} of pairs (S, d) where S is a nonempty subset of n -dimensional real linear space \mathbb{R}^n and d is a point in \mathbb{R}^n .

There are two basic approaches to dealing with these problems in the framework of game theory: the strategic one and axiomatic one. In the strategic approach, the outcome is an equilibrium in an explicit model of the bargaining process (usually a game in extensive form). In the axiomatic approach the outcome is defined by a list of properties that it is required to satisfy (for example, individual rationality, Pareto optimality, symmetry).

First we briefly discuss relation between the two most famous models of two-player bargaining; namely, the Nash axiomatic model [2], and the Rubinstein strategic model of alternating offers [7], [4]. Then we focus our attention to the so called sequential or stepwise solutions of axiomatic models [1]. Finally, we propose some open problems for future research.

Key words

Negotiation, bargaining, arbitration, stepwise solutions.

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Robustness Analysis of Road Networks

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Abstract

The main subject of the presentation is to introduce new methods which enable us to measure robustness of road networks. We show how to evaluate susceptibility of the road networks to varied types of random attacks. The process of evaluation is based upon analysis of consequences of random processes comprising, among other things, suitable modifications of Hard-Core model.

Key words

Robustness, road networks, random processes.

A Note on Computing Extreme Tail Probabilities of the Noncentral T Distribution with Large Noncentrality Parameter

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Abstract

The central as well as the noncentral t distribution belong to the most frequently used distributions in statistics. The cumulative distribution function (CDF) of noncentral t distribution, say $F_{t_{\nu,\delta}}(\cdot)$, gives the probability that a t test will correctly reject a false null hypothesis on mean of a normal population, i.e. the test of the null hypothesis $H_0: \delta = \delta_0$ against the alternative $H_A: \delta > \delta_A$ (based on small sample from this population), when the population mean is actually δ_A ; that is, it gives the power of the t test. Broad applicability of the noncentral t distribution is also in engineering, measurement science and metrology. For example, it is used for calculating the endpoints of the one-sided tolerance intervals (the tolerance limits) for a normal population.

Here we compare methods and algorithms for computing extreme tail probabilities of the noncentral t distribution with possibly very large noncentrality parameter δ and/or degrees of freedom ν . As we shall illustrate (in such cases), application of simple numerical quadrature,

$$F_{t_{\nu,\delta}}(x) = \Phi(-\delta) + \int_{-\delta}^{\infty} \left(1 - F_{\chi_{\nu}^2} \left(\frac{\nu(z + \delta)^2}{x^2} \right) \right) \phi(z) dz, \quad (8)$$

can be more precise and efficient, if compared to the more advanced implementations, see e.g. [1], [2]. Equation (8) holds true for $x > 0$, where $\Phi(\cdot)$, $\phi(\cdot)$, and $F_{\chi_{\nu}^2}(\cdot)$ denote the CDF/PDF of standard normal distribution, and CDF of chi-square distribution with ν degrees of freedom, respectively.

Key words

Noncentral t distribution, computing extreme tail probabilities.

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On the Likelihood Based on Fuzzy Data

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Abstract

The likelihood for parametric continuous models usually is based on the joint density of samples. In case of fuzzy data, it is possible to base it on the probabilities of the observed fuzzy data because the fuzzy observations have positive probabilities. Based on this, it is possible to propose a more natural definition of likelihood functions. This allows to construct maximum likelihood estimation also in case of fuzzy data.

Key words

Fuzzy numbers, characterizing function, likelihood function.

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Incorrectly Posed Systems of (max, min)-Linear Equations and Inequalities

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Abstract

We consider the following system of equations and inequalities:

$$\max_{j \in J} (a_{ij} \wedge x_j) = b_i, \quad i \in I, \quad \underline{x} \leq x \leq \bar{x},$$

where operation \wedge denotes “min”, i.e. $\alpha \wedge \beta = \min(\alpha, \beta)$, a_{ij}, b_i are real numbers, I, J are finite index sets, \underline{x}, \bar{x} are given lower and upper bounds of variables x . Equations of the system are called (max, min)-linear equations. Theoretical results and other applications concerning similar algebraic structures can be found in the literature.

Properties of the systems as well as their possible applications to some fuzzy sets and operations research problems will be surveyed. Main subject of the contribution is devoted to situations, in which the given system has no solution and it is necessary to find a solvable system of (max, min)-linear equations, which is in some sense close to the original unsolvable system. Several approaches to finding close solvable systems will be proposed. Perspectives of further research will be briefly discussed.

Key words

(max,min)-linear equation, close solvable systems, operations research problems, fuzzy sets.

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Computing the Greatest Common Divisor of Polynomials

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Abstract

Euclid's algorithm and the reduction of the Sylvester resultant matrix to upper or lower triangular form are two common well known methods for computing the greatest common divisor (GCD) of two univariate polynomials. It is firstly supposed that the coefficients of polynomials are given exactly. Consider the polynomials f_0 and f_1 ,

$$\begin{aligned} f_0(x) &= a_0x^{n_0} + a_1x^{n_0-1} + \cdots + a_{n_0-1}x + a_{n_0}, & a_0 a_{n_0} &\neq 0, \\ f_1(x) &= b_0x^{n_1} + b_1x^{n_1-1} + \cdots + b_{n_1-1}x + b_{n_1}, & b_0 b_{n_1} &\neq 0, \end{aligned}$$

respectively, where it is assumed without loss of generality that $n_0 \geq n_1$. The Sylvester matrix $S(f_0, f_1)$ of f_0 and f_1 is introduced in the lecture. The transformations of the Sylvester subresultant matrix are described in details and quantitative relation between the rank of the Sylvester subresultant matrix and the degree of GCD is formulated. The algorithm for calculation of GCD of two polynomials is presented.

For polynomials with perturbed coefficients an approximate greatest common divisor (AGCD) is defined and calculations with inexact polynomials are shortly discussed. A numerical example concludes the lecture.

Key words

Sylvester matrix, Euclid's algorithm, greatest common divisor.

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Numerical Comparison of some Algorithms for the Vehicle Routing Problem

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Abstract

The vehicle routing problem (VRP) is an important problem in operational research and integer programming. This problem has practical applications especially in logistic. VRP problem can be described as the problem of finding optimal routes for fleet of vehicles from one or several depots to a number of customers. The goal is usually to find the minimal cost of transportation. There are many variants of this problem which differ other prescribed conditions, for example vehicle routing problem with time windows or capacitated vehicle routing problem.

There exist some exact algorithms for solution of this problem, but finding global minimum is computationally complex, because problem is NP-hard. Therefore some approximate algorithms are often used that seek only sufficiently good solution. We give numerical comparison of some selected algorithms on some problems of different sizes.

Key words

Vehicle routing problem, exact algorithm, heuristic.

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