

ODAM 2011

The international conference

Olomoucian
Days
of
Applied
Mathematics

Book of Abstracts

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Faculty of Science
Palacký University in Olomouc**

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Preface

The conference “Olomoucian Days of Applied Mathematics” (ODAM) came into existence through the initiative of Professor Lubomír Kubáček in 1999 in relation to the seminar of applied mathematics held by the Department of Mathematical Analysis and Applications of Mathematics at Faculty of Science of Palacký University in Olomouc, whose focus and topics were forged in cooperation with researchers from Mathematical Institute of Academy of Science of Czech Republic, Faculty of Mathematics and Physics at Charles University in Prague, and Hydrosystem company. From the very beginning, the conference was conceived as a friendly meeting of experts in mathematical applications, which, over time, gave it a twofold focus—on mathematical statistics and fuzzy sets on the one hand, and mathematical modeling on the other—these two taking turns annually. For the first time, the conference O DAM 2011 will include all the fields of applied mathematics currently cultivated at the Department of Mathematical Analysis and Applications of Mathematics, i.e., mathematical and applied statistics, fuzzy sets and their applications, methods of multiple-criteria evaluation and decision making, numerical mathematics, and partial differential equations. Moreover, this year witnesses a transformation of a traditional Czecho-Slovak meeting into an international conference with English as a working language.

ODAM 2011 commemorates an important anniversary in the life of Prof. Kubáček—one of the leading figures of Czech and Slovak mathematical statistics. On this important occasion, the organizers intend to launch a new series of international conferences on applied mathematics in Olomouc.

I very much appreciate that the financial support from the grant “Streamlining the Applied Mathematics Studies at Faculty of Science of Palacký University in Olomouc” (MAPLIMAT, in short) made it possible to organize the O DAM 2011 conference in this new form and in the very honor of Prof Kubáček who has contributed enormously to developments in applied mathematics at this Faculty. We are all very pleased that outstanding experts on statistics, fuzzy sets, and numerical mathematics accepted our invitation to give their talks at the conference; and our big wish is also their future participation. In line with Prof Kubáček’s original idea, we will strive to shape the O DAM conference as an opportunity for young researchers and post-graduate students to meet with leading figures of applied mathematics, and to gain experience with presenting their own research. As a part of the above-mentioned project, the O DAM conference has one special feature—from now on, attendance at the conference is included as an optional subject in the study schedules of all the fields of applied mathematics being streamlined under this project; that is, even pre-graduate students have the opportunity to attend talks of outstanding experts and the presentations of invited lectures will be made available to them as study materials on the project’s website.

I wish all the participants of the O DAM 2011 conference days pleasantly spent in our beautiful historical city of Olomouc!

To conclude, I would like to extend my thanks to all members of the organizing team and to those students who helped before and during the conference.

Jana Talašová
Chief Manager of MAPLIMAT Project

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Linear Regression with Interval Data: Computational Issues

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Abstract

Some attention has been recently paid to the analysis of interval data, which are more and more often encountered in the practice. For example, financial data have bid-ask spreads, demographic or epidemiologic data are typically available only in an interval-censored form because exact individual characteristics are unpublished. Analogously, some types of categorical data may be also regarded as interval data as, e.g., credit rating grades, which are sometimes regarded as intervals of credit spreads over the risk-free yield curve. There also exists one very important application in the theory of computation. If we store data using data types of restricted precision, then instead of exact values we are only guaranteed that the true value is in an interval of width 2^{-d} , where d is the number of bits of the data type for representation of the non-integer part.

In the lecture we will deal with linear regression models with interval data. The main part of our lecture will be devoted to the models with interval observations for dependent variables. However, for the sake of completeness, we shall also comment on models with both dependent and independent variables of interval type.

Interactions among Criteria as Modelled by Fuzzified Choquet Integral

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Abstract

The paper deals with fuzzification of Choquet integral along with the employment of Choquet integral and its fuzzified forms in models of multiple criteria evaluation. In the first part of the paper, we study a role of the discrete Choquet integral [3] in models of multiple criteria evaluation. We focus on the generalisation of Partial Goals Method (PGM) [2] in order to overcome difficulties arising from interdependency among criteria when special types of interaction, e.g. redundancy and complementarity, occur. The proposed PGM generalisation begins with the change of aggregation operator. The original aggregation operator based on an additive measure, weighted average, is replaced by Choquet integral that employs a generalised monotonous measure—a fuzzy measure. We study general conditions for application of Choquet integral to the aggregation of partial evaluations in the generalized PGM. Moreover, we present some examples of decision making problems, where the criteria are dependent, and yet the use of Choquet integral is not advisable. In the second part, the generalisation of PGM proceeds with fuzzification of Choquet integral [1]. We propose effective ways of setting the fuzzified fuzzy measure on the set of partial goals. Both fuzzy versions of the generalised PGM are explained through examples.

Key words

Aggregation, Choquet integral, fuzzification, evaluation models.

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Rate of Return in Building Savings

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Abstract

Building savings is a financial product that enables a participant to finance housing. A participant saves money for a certain period and so claims to an advantageous credit. The advantage of the credit compared to a mortgage loan consists in the relatively fast administration dispatching with the relatively low costs. During the saving a participant gains in addition to tax-free interests the subsidy depending on yearly deposited sum. The subsidy has made 15 % of the deposited sum, but it is limited to 3 000 crowns now and it is going to decrease in the future. There are several possibilities, how to save money. A participant usually saves the money monthly, but yearly or irregular deposits or even a lump sum are also permitted. Rate of return on building saving is expected to be depended on the saving frequency and other factors. Analysis of rate of return in building saving is the aim of the contribution. Relative and net relative current return, annual and net annual rate of return and effective interest rate will be shown in the contribution. The subsidy 15 % of the deposited sum, limited to 3 000 crowns, is assumed. The calculations of the named rates return will be also performed under new conditions, concerned the sum of the subsidy, that the Czech government has suggested. Obtained results will be compared to the rate of return of selected saving account.

Key words

Building savings, rate of return.

References

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Thoughts about Selected Models for the Valuation of Real Options

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Abstract

Real options are the different types of managerial flexibility found in connection with real investments that allow managers to capture the potential in the investment. Investments with real options are more valuable than the same investments without real options, *ceteris paribus*. This is why it is of interest to many stakeholders to be able to value real options.

The term real options is based on the observation that managerial flexibility found in real investments exhibits an analogy of construct with financial options, see e.g. [6]. The original idea with real option valuation is to use models created for financial option pricing in the valuation of real options. After Black & Scholes 1973 article [1] on pricing of options a model for option valuation was available and the road for the valuation of real options was open. As new methods for option valuation were developed, i.e. the binomial option valuation model [4], they were also used in the valuation of real options.

Lately the paradigm of using financial option valuation models to real option valuation has been seriously questioned [2] and more focus is shifting towards the new real option valuation methods that have been created to better suit the needs of real option valuation and managers. This paper discusses option valuation logic and two of the new methods the Datar–Mathews method [5, 7] and the Fuzzy Pay-off Method [3] in together with the Black & Scholes and the Binomial methods to understand the big picture of practical real option valuation today.

Key words

Real option valuation, option valuation models.

References

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Optimal Energy Transport by Advection

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Abstract

Let us consider the boundary value problem

$$-\varepsilon y'' + v y' + \alpha y = \alpha y_M \text{ in } (a, b), \quad y(a) = y_M, \quad v(b)y'(b) + \alpha y(b) = \alpha y_m \quad (1)$$

with the constant coefficients $\varepsilon > 0$, $y_m < y_M$ and $\alpha > 0$. This contribution is devoted to the following problem: For a given positive constant C , find a positive continuous function v such that

$$\int_a^b v(x) dx = C$$

and the value $y(b)$ of the solution y of the problem (1) is maximal.

Problem (1) is a simple rough model of the heat-transport inside of a medium flowing through the interval $\langle a, b \rangle$ with velocity v . The medium enters the interval $\langle a, b \rangle$ with a high temperature y_M and, during the flow, the heat permeates from the medium into the environment with low temperature y_m . The aim of our analysis is to find such a distribution $v(x)$ of the velocity with a given average $C/(b - a)$ that the loss of heat during the flow through the interval $\langle a, b \rangle$ is minimal.

By describing the solution of this problem, we show that the amount of the loss of energy does depend on the concrete distribution of velocity in the real systems exploiting the advective energy transport and indicate the form of this dependence.

Thanks

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Compositional Aspects of Orthogonal Regression

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Abstract

Compositional data as a kind of multivariate data, carrying only relative information in their parts, require different treatment in sense of accomplishing standard statistical techniques. In the contribution we will focus on orthogonal regression also known as errors-in-variables modeling, total least squares technique or calibration line problem solved using linear statistical models with type-II constraints [1]. Initially, in order to be able to perform orthogonal regression for compositional data we should use isometric log-ratio (ilr) transformation that maps the compositions to standard multivariate observations. The ilr transformation is an isometric mapping between the compositional data sample space, the simplex S^D , and the Euclidean real space \mathbb{R}^{D-1} and corresponds to coordinates of an orthonormal basis on the simplex. Improving interpretation of ilr coordinates can be achieved by finding an appropriate orthonormal basis. In addition, under the assumption of normality of the ilr transformed data, which implies normality on the simplex as well, the linear models' approach to orthogonal regression enables to construct confidence regions, test hypothesis and also allows to solve many others tasks from the field of linear models. The main goal of this contribution is to analyse results from orthogonal regression for three-part compositions, that leads to fitting a line for n two-dimensional data, so that the sum of the squared distance from the data points to the estimated calibration line would be minimal. In particular, we will focus on the results using interpretation of the ilr coordinates. They can be approached as balances between groups of parts of a composition, constructed by sequential binary partition, as well as by expressing their covariance structure by log-ratios of parts of the analyzed composition, i.e. in terms of ratios. The above mentioned theoretical considerations will be illustrated on real-world examples.

Key words

Isometric log-ratio transformation, linear regression model, calibration line, hypotheses testing, confidence bounds.

References

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Thanks

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The Hilbert Space of Density Functions

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Abstract

The statistical analysis of compositional data, based on the approach put forward by John Aitchison, suggested the definition of operations and a metric in the simplex, which define an Euclidean space structure. The elements of the simplex are interpreted as representatives of equivalence classes of vectors with proportional positive components. A typical example of a composition is a histogram of frequencies, which describes a discrete distribution of probability. Within this structure, the Abelian group operation, or *perturbation*, is identified with Bayes formula. The dimension of the simplex as vector space is the number of categories or parts minus one.

This fact suggests that, increasing the number of parts of a simplex towards infinity, and consequently its dimension, it will approach a Hilbert space which contains the probability densities with continuous support. A first result, specific for distributions with support an interval of the real line, was developed in [1]. The measures with densities which square logarithm is integrable have a Hilbert space structure. This space includes finite measures, equivalent to probability measures, and also infinite measures, which are equivalent to improper densities used in Bayesian statistics (improper *priors* or likelihoods). The space structure allows the introduction of a distance, orthogonal projections, Fourier developments on Hilbert bases, etc. Recently, the generalization of these results to unlimited support has been obtained. The concept of support of a measure is replaced by a reference measure. The resulting Hilbert space of measures allows the use of Aitchison type distances and projections for general probability densities.

Key words

Compositional data, simplex, Euclidean space.

References

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Multiobjective De Novo Linear Programming

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Abstract

Mathematical programming under multiple objectives has emerged as a powerful tool to assist in the process of searching for decisions which best satisfy a multitude of conflicting objectives. There are a number of distinct methodologies for multicriteria decision-making problems that exist. Multiobjective linear programming (MOLP) problem can be described as follows

$$\begin{aligned} & \text{Max } Cx \\ & \text{s.t. } Ax \leq b \\ & \quad x \geq 0 \end{aligned} \tag{1}$$

where C is a (k, n) -matrix of objective coefficients, A is a (m, n) -matrix of structural coefficients, b is an m -vector of known resource restrictions, x is an n -vector of decision variables. In MOLP problems it is usually impossible to optimize all objectives in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing this for another objective. Trade-offs are properties of inadequately designed system a thus can be eliminated through designing better one. Multiobjective De Novo linear programming (MODNLP) is problem for designing optimal system by reshaping the feasible set. By given prices of resources and the given budget the MOLP problem (1) is reformulated in the MODNLP problem (2)

$$\begin{aligned} & \text{Max } Cx \\ & \text{s.t. } Ax - b \leq 0 \\ & \quad pb \leq B, \quad x \geq 0 \end{aligned} \tag{2}$$

where b is an m -vector of unknown resource restrictions, p is an m -vector of resource prices, and B is the given total available budget. The paper presents approaches for solving the MODNLP problem, extensions of the problem, examples, and applications.

Key words

De Novo Programming, multiple objectives, liner programming, trade-offs.

References

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Thanks

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Discriminant Analysis for Compositional Data and Robust Parameter Estimation

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Abstract

Compositional data, i.e. data including only relative information, need to be transformed prior to applying the standard discriminant analysis methods that are designed for the Euclidean space. Here it is investigated for linear, quadratic, and Fisher discriminant analysis, which of the transformations lead to invariance of the resulting discriminant rules. Moreover, it is shown that for robust parameter estimation not only an appropriate transformation, but also affine equivariant estimators of location and covariance are needed. An example and simulated data demonstrate the effects of working in an inappropriate space for discriminant analysis.

Key words

Compositional data, logratio transformations, discriminant analysis, outliers.

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Various Approaches to Orthogonal Regression

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Abstract

Orthogonal regression is a proper tool for fitting two-dimensional data points when errors occur in both the variables. This type of modelling technique is also called the total least squares (TLS) in the statistical literature. In its simplest form it attempts to fit a line that explains the set of n two-dimensional data points in such a way that the sum of squared distances from data points to the estimated line is minimal.

The difficulty or even impossibility of deeper statistical analysis (confidence regions, hypotheses testing) using the standard solution for orthogonal regression based on maximum-likelihood method can be overcome by calibration line technique based on linear statistical models, namely linear models with type-II constraints (constraints involve in addition to the unknown model's parameters the other unobservable ones). The main advantage of the linear model approach is its validity for finite samples in contrast to the standard techniques. It means we can determine exact variances and covariances of estimated line's coefficients (for the standard technique we have only asymptotic variances and covariances). Further, under assumption of normality, we can make any standard statistical inference, e.g., construct confidence regions and bounds and test hypotheses. Consequently, we can apply various standard approaches to checking the model and its assumptions for adequacy and validity, e.g. coefficient of determination, residuals analysis or normality tests.

The aim of the contribution is to present three various approaches to orthogonal regression. Particularly, two standard solutions based on maximum-likelihood or on singular value decomposition methods and an iterative algorithm for estimating the regression line via linear models with type-II constraints. Further, we will present some statistical inference. The theoretical results will be applied to real-world example.

Key words

Orthogonal regression, total least squares, linear model with the type-II constraints, calibration line.

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Automatic Analysis of Digital Images Originating in Quantitative Immunofluorescent Analysis of DNA Damage Markers

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Abstract

This contribution deals with the problem of automatic digital image analysis. The problem is a part of an Interior Ministry Project aimed at a proposal of new methods for evaluating cell exposition level to ionizing radiation. The images to be analyzed originate in microscopic analysis of tissue that had been irradiated by cobalt gamma source. The extent of DNA damage, which is to be assessed, is proportional to the signal from fluorescent foci which appear as bright spots inside the cells in the image. Each focus marks a modified histone at the point of a DNA fault, where a fluorescent marker is immunochemically attached.

The image analysis itself has three phases: First, individual cells in the image are recognized. This is achieved by means of a standard adaptive threshold algorithm. Next, it is necessary to assess which of the recognized objects represent several touching cells, rather than a single one. This is an interesting problem and three approaches are possible: (1) Multiple objects can be detected by means of the distance transform and then separated by a watershed algorithm. (2) Another possibility is to perform segmentation of the object boundary and then use ellipse fitting techniques. (3) The third approach exploits neural network algorithms. A combination of the three approaches may be necessary, accompanied by a heuristic which uses a-priori knowledge of the image particular form. The last stage in the image analysis consists of statistical evaluation of the foci signal, which appears to be straight-forward.

Key words

Automatic analysis of digital images, immunofluorescent analysis, DNA damage markers.

References

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On the Incompatibility of Richards' Equation and Finger-Like Infiltration in Unsaturated Homogeneous Porous Media

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Abstract

It was demonstrated by means of a mathematical proof that Richards' Equation, in principle, cannot admit finger-like solutions for three-dimensional homogeneous unsaturated porous media flow, subject to monotone boundary conditions. This was demonstrated for reasonable type of homogeneous porous material—the result is not dependent on any particular form of the hydraulic conductivity or the retention curve. Moreover, it was explained why hysteresis of the retention curve does not play any role in the proof. Consequently, the proof is true for any type of hysteretic behavior of the retention curve.

Key words

Richard's equation, finger-like solutions.

References

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Thanks

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Algorithms for Uniform Sampling from n -spheres and n -balls

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Abstract

Sampling from the uniform distribution on the n -dimensional Euclidean ball (the n -ball) and its surface (the n -sphere) is a tool useful, e.g., for Monte Carlo integration, generating random directions, rotations and correlation matrices.

There are essentially three approaches to sampling from the uniform distribution on the n -ball and the n -sphere. The first approach to generating from the uniform distribution on the n -ball and, by a normalization, on the n -sphere, is using the rejection method on an n -cube circumscribing the n -ball. However, for $n \rightarrow \infty$, the ratio of accepted and rejected samples decreases to 0 extremely fast, which means that the rejection method is unusable for higher dimensions.

The second approach uses the fact that the multivariate normal distribution with the unit covariance matrix is radially symmetric. Therefore, if $\mathbf{X} \sim N_n(\mathbf{0}_n, \mathbf{I}_n)$, then $\mathbf{S}_n = \mathbf{X}/\|\mathbf{X}\|$ has the uniform distribution on the unit n -sphere. If we multiply \mathbf{S}_n by $U^{1/n}$, where U has the uniform distribution on $(0, 1)$, we obtain the uniform distribution on the unit n -ball.

The algorithms in the third group utilize the properties of the marginals of the uniform distribution on n -spheres and n -balls, see, for instance, [1] and [2]. In the talk, we will describe a unified approach based on a family of inter-related multivariate distributions on n -balls that generalizes these algorithms and clarifies their mutual relations. Moreover, we will propose new methods for generating uniformly from n -balls and n -spheres which, in some cases, turn out to be the best available. The talk is based on the paper [3].

Key words

Monte Carlo, n -Spheres, n -Balls, uniform distribution.

References

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Thanks

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Free Material Optimization

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Abstract

Material optimization is a special branch of structural optimization in which the behavior of structures is controlled by an appropriate choice of materials from which these structures are made. The respective mathematical models lead to optimal control problems with control variables appearing in coefficients of state problems, usually given by partial differential equations. In the case of linear elasticity such variables are represented by coefficients of a linear Hook law. These problems are not (very often) well-posed since they have no solution. To ensure the existence of a solution one has to enlarge the system of admissible controls (materials). One way how to do that is to use a homogenization approach when the admissible set of materials is enriched by all possible mixtures of a-priori given constituents. Another way is to use the so-called free material approach which is the subject of this presentation. Unlike to the homogenization approach the admissible set of coefficients now covers all physically admissible materials (coefficients of the 4th order positively semi-definite tensors in linear Hooke's, in our case). We prove that with such a choice of the admissible set the problem has a solution, i.e. the problem is well-posed. The second part of this contribution is devoted to a discretization of free material optimization problems and convergence analysis. Finally numerical results of model examples will be shown.

The presented results were obtained jointly with M.Kočvara (University of Birmingham), M. Leugering and M. Stingl (University of Nürnberg-Erlangen).

Key words

Free material optimization, H-convergence, semi-definite programming.

References

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Thanks

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Combination of 3D Epoch-Wise and Permanent Geodetic Networks Observed by Global Navigation Satellite Systems

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Abstract

The local, regional and global geodetic reference networks are recently almost exclusively observed by satellite radionavigation methods, such as the U.S. Global Positioning System, Russian navigation system GLONASS, and the upcoming European system Galileo. The unprecedented accuracy of satellite methods allow determination of site coordinates at millimetre level. We will introduce the complex adjustment model for combination of 3D coordinates observed in permanent and epoch-wise satellite networks. Estimated parameters include the site coordinates, site velocities reflecting the dynamics of the Earth's crust, transformation parameters among reference frames and other additional unknowns, like apparent station shift due to observing equipment changes, etc. The combination procedures require relevant stochastic modelling of observation noise, the models of coloured noise will be introduced. The applied methods will be demonstrated on local and regional GPS networks in Slovakia and in Central Europe.

Key words

3D Global Navigation Satellite System Network, adjustment model for combination of 3D coordinates.

References

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The Software Support for Fuzzy Multiple-Criteria Evaluation

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Abstract

The software product FuzzME was developed as a tool for creating fuzzy models of multiple-criteria evaluation and decision making. The type of evaluation employed in the fuzzy models fully agrees with the paradigm of the fuzzy set theory; the evaluations express the (fuzzy) degrees of fulfillment of corresponding goal. FuzzME allows the utilization of several aggregation methods—fuzzy weighted average, fuzzy OWA operator, fuzzified WOWA operator, fuzzified discrete Choquet integral, and fuzzy expert system. These methods will be described and an illustrative example of their application from the area of banking will be given.

Key words

Multiple-criteria evaluation, fuzzy methods, criteria interactions, software.

References

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Relative Influence of Copula and Marginals on Performance of Calibration Methods

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Abstract

Different calibration methods are investigated for data simulated from various copula classes and marginals under both random and controlled calibration setup.

Key words

Calibration, copula, posterior, likelihood.

References

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Multivariate Kernel Density Estimates

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Abstract

Multivariate kernel density estimates can be considered as an important non-parametric tool for exploratory data analysis. The need for kernel density estimates for recovering structure in multivariate data is greater since parametric modelling is more difficult than in the univariate case. The most important factor in multivariate kernel density estimates is a choice of bandwidth matrix H . Because of its role in controlling both the amount and direction of multivariate smoothing this choice is particularly important. Most of popular bandwidth selection methods in a univariate case (see e.g., Scott 1992, Wand, Jones 1995) can be transferred into multivariate settings (Duong, Hazelton 2005, Chacón, Duong 2009). The problem of the bandwidth matrix selection can be simplified by imposing constraints on this matrix. A common approach to the multivariate smoothing is to first rescale the data so the sample variance are equal in each dimension—this approach is called scaling or sphering the data so the sample covariance matrix is the identity matrix. Provided that the bandwidth matrix is diagonal the method for its choice in bivariate case without using any pre-transformations of the data has been proposed in paper Horová, Koláček, Vopatová (2010). In this case there exists the explicit solution of the corresponding minimization problem, i.e. $H^* = \operatorname{argmin} \operatorname{AMISE}(H)$. This fact enable us to find useful relation between the asymptotic integrated variance and the asymptotic integrated square bias, which is a basis of the developed method. In the present paper we focus on generalization of this method both in direction of d -variate density and a full bandwidth matrix. Moreover, since significant information about features of the density is contained in the first derivative of the true density we also pay attention to the kernel gradient estimator. We conduct a simulation study comparing the least square cross-validation method and the proposed method.

Key words

Kernel, bandwidth matrix, asymptotic mean integrated square error.

References

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Thanks

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On the Interpretation of Orthonormal Coordinates for Compositional Data with Applications

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Abstract

A natural sample space for compositional data, i.e., observations carrying only relative information (especially proportions, percentages, etc.), is the simplex with the Aitchison geometry [1,2]. For this reason, standard statistical methods that rely on Euclidean structure of the real space cannot be used directly for this kind of data. At first, compositional data need to be expressed in coordinates of an orthonormal basis on the simplex (with respect to the Aitchison geometry). The mathematical interpretation of the orthonormal coordinates is derived from the procedure of their construction, called sequential binary partition, as balances between groups of compositional parts [2]. The main goal of the contribution is to describe the covariance structure of coordinates and, consequently, to provide a complementary interpretation based on log-ratios of parts of the original composition [3]; note that in a composition the ratios themselves contain all the relevant information. Additionally, we show how this approach can be used for linear regression of compositional data.

Key words

Aitchison geometry on the simplex, orthonormal coordinates, regression analysis.

References

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AHP Model for Ranking of Efficient Units in DEA Models

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Abstract

Traditional data envelopment analysis (DEA) models are not able discriminate among efficient decision making units (DMUs) because all of them have maximum efficiency score 100 %. That is why several modelling approaches for complete ranking of DMUs including efficient ones were proposed. Super-efficiency models which are based on removing the evaluated DMU from the set of units and measuring a distance of this unit from the new efficient frontier belong to most popular approaches by practitioners and researchers. Other methodological approaches are based on cross-efficiency evaluation, optimistic and pessimistic measuring the efficiency, the benchmark method, and several statistical techniques. Several attempts using principles of multiple criteria decision making were published as well.

The aim of the paper is to present an original approach for ranking of efficient DMUs based on the analytic hierarchy process (AHP) developed by T. Saaty. AHP organises decision problems as hierarchical structures containing always several levels. The topmost level defines the goal of the decision problem which is ranking of the DMUs in our case. The next level contains decision criteria (inputs and outputs). This level can be structured into several particular sub-levels. The lowest level of the hierarchy describes alternatives (DMUs). The DMUs cannot be pairwise compared with respect to inputs and/or outputs directly because of possible significant differences among DMUs in their size and input and output values. That is why we suggest comparing them with respect to all ratios outputs/inputs which describe particular efficiencies of the DMUs. Depending on the number of efficient DMUs that are ranked either the AHP with relative measurement or absolute measurement can be used. The approach runs in two basic steps. The first one is traditional DEA analysis and specification of efficient DMUs. In the second step the AHP model is created with second hierarchical level containing all ratios outputs/inputs. Their priorities are derived as average weights from DEA analysis. Finally the DMUs are evaluated with respect to all criteria and their priorities are derived. The priorities generate complete ranking of DMUs. The proposed approach is illustrated on a numerical example with a real-world background. The results of the DEA/AHP model are compared with other approaches.

Key words

Data envelopment analysis, analytic hierarchy process, efficiency, super-efficiency.

Thanks

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Trimmed Estimators in Regression Framework

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Abstract

From practical point of view the regression analysis and its Least Squares method is clearly one of the most used techniques of statistics. Unfortunately, if there is some problem present in the data (for example contamination), classical methods are not longer suitable. A lot of methods have been proposed to overcome these problematic situations. In this contribution we focus on special kind of methods based on trimming. There exist several approaches which use trimming off part of the observations, namely well known high breakdown point method the Least Trimmed Squares (presented in [1]) or regression L-estimate Trimmed LSE of Koenker and Bassett [2] or for example recently proposed the Least Trimmed Quantile Regression [3]. Our goal is to compare these methods and its properties in detail.

Key words

Least trimmed squares, trimmed LSE, regression quantiles.

References

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Automotive Lighting with Improved Luminous Homogeneity

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Abstract

The strict competition forces the carmakers to hold customers' interest in many ways. The mass oncoming of light emitted diodes (LEDs) as new light sources into the automotive industry gives so desired opportunities and enables both to enhance design potentialities and to decrease power consumption. The carmakers demand competitive price of the components, novel technical solutions and tuned optical properties, namely optical efficiency, luminous homogeneity and lit appearance. The homogeneity requirements start to be essential for signal lighting since the front and rear position lights, direction indicator lamps and daytime running lights are used to emphasize exterior styling features of motor vehicles. The homogeneity of illumination of dashboards and other interior parts is a crucial parameter as well.

The optical systems used by suppliers usually contain light source(s) and reflecting and refracting surfaces including reflectors, lens, filters, prisms, light pipes and collimators. The optical components are usually 3D objects bounded by base spline surfaces. Despite the possible simplifications a homogeneity problem often leads to a nonlinear equation describing the geometrical situation. We mention different notions of homogeneity, the homogeneous lit appearance and the homogeneity of the illumination of an area.

Key words

Automotive lighting, signal lighting, ray optics, homogeneous illumination, LED.

References

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Least Weighted Squares Applied to Robust Image Analysis

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Abstract

The paper is devoted to robust statistical methods with applications to image analysis. The idea of the least weighted squares (LWS) estimator down-weighting less reliable observations is applied in different situations to propose new robust estimation procedures with a high breakdown point. The methods are illustrated on a database of two-dimensional grey-scale images of faces.

The LWS method is studied in the location model and applied to image denoising. We also propose a robust alternative to Moran's autocorrelation coefficient, which allows to measure autocorrelation in images in a robust way.

Finally we propose the minimum weighted covariance determinant (MWCD) estimator as a weighted analogy of the minimum covariance determinant (MCD) estimator for the multivariate model. We study its robustness properties, describe an approximative algorithm for its computation and demonstrate its performance and robustness in a simulation study. While the MCD estimator suffers from local sensitivity, the new estimator is more resistant to small changes of the data. The performance of the estimator is illustrated on a classification problem of localizing the mouth in images of faces.

Key words

Least weighted squares, robust estimation, multivariate statistics, high dimension.

References

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Evaluation of Granules of a Fuzzy Relation by Integrals

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Abstract

We will consider two universes, U and V and an arbitrary fuzzy relation $R: U \times V \rightarrow [0, 1]$. A pair of sets $(X, Y) \in 2^U \times 2^V$ induces a granule R_{XY} of R (which is R restricted to $X \times Y$). A numerical evaluation of granule R_{XY} gives information about the relationship between X and Y . In [1] some evaluations based on aggregation of membership grades of R_{XY} by two aggregation functions were examined. In [2] the representation of R by α -level sets R_α (for $\alpha \in (0, 1]$) was discussed and lower and upper bounds for approximation of $Y \subset V$ given by

$$\underline{R}_{\alpha, Y} = \{x \in U; r_\alpha \subset U\}, \quad \bar{R}_{\alpha, Y} = \{x \in U; r_\alpha \cap U \neq \emptyset\}, \quad (1)$$

where

$$r_\alpha(x) = \{y \in V; R(x, y) \geq \alpha\}, \quad (2)$$

were proposed.

In this contribution we will focus on evaluation of the relationship between $X \subset U$ and $Y \subset V$ by integrals. Several types of integrals with respect to monotone set functions will be considered and illustrated with examples. For basic information on integrals with respect to monotone set functions we refer to [3].

Key words

Granule, evaluator, fuzzy relation, aggregation function, integral.

References

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Stochastic Modelling of Neuron Activity

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Abstract

The special case of spatio-temporal Cox processes constructed from Lévy basis is studied. Formulas for theoretical characteristics are derived using the generating functional. The Cox process on the curve is defined and studied. The analysis of such a process leads to nonlinear filtering methods. These methods are used on the real data from a neurophysiology experiment. During the experiment, the spiking activity of a place cell of hippocampus of a rat moving in an arena together with the track of the rat was recorded. The track of the rat and the action potentials (spikes) present the curve and the points on it. Also other approaches to neurophysiological data are discussed. The first one is an estimation of a conditional intensity of the temporal process of spikes using recursive filtering. In the second one, the track of the rat together with the random driving intensity function of the process of the spikes is viewed as a random marked set.

Key words

Cox point process, spatio-temporal process, random marked closed.

References

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The Type A Uncertainty

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Abstract

If in the model of measurement except useful parameters, which are to be determined, other parameters occur as well, which were estimated from another experiment, then the type A and B uncertainties of measurement results must be taken into account. The type A uncertainty is caused by the new experiment and the type B uncertainty characterizes an accuracy of the parameters which must be used in estimation of useful parameters. The problem is to estimate of the type A uncertainty in the case that the type B uncertainty is known.

Key words

Two stage linear model, the type A and B uncertainties.

References

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Comments to the Lineweaver–Burke Transformation

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Abstract

If the Lineweaver–Burke transformation [3] is used, a linear regression model is obtained and the standard least-squares procedure [5] can be used.

The problem is to find statistical properties of the estimators.

Another approach for an estimation of the parameters β_1 and β_2 is the linearization. This approach can be used only if the measure of nonlinearity [1], [4] is sufficiently small.

It is interesting to compare the estimators based on the Lineweaver–Burke transformation and on the linearization in a simulation study.

Key words

Linearization, weakly nonlinear regression model, measure of nonlinearity.

References

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On Optimization Algorithms for Solving Contact Problems

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Abstract

The contribution deals with solving contact problems of linear elasticity with friction using optimization algorithms. The starting point is the algebraic form of the problem arising from the finite element approximation. It consists in a linear equality (inner equilibria) and a system of non-linear inequalities (complementarities describing the contact conditions). The solution to the Coulomb model of friction may be defined as a fixed point of a mapping that is described by an auxiliary problem with the Tresca model of friction. Its dual formulation leads to the minimization of a strictly quadratic objective function constrained by simple bounds and quadratic inequalities. There are three principal iterative concepts how to solve such minimizations: strictly feasible, feasible and infeasible. The typical representant of the feasible algorithm is the active set method that combines the conjugate gradient iterations with the projection steps [1]. Unfortunately, this algorithm generates “zig-zag” iterations, i.e., it switches many times between short conjugate gradient sequences and projection steps in order to change the active set. The interior point method [2] represents the strictly feasible strategy. It excludes the usage of the active set so that the “zig-zag” phenomena is suppressed. Finally, the semi-smooth Newton method may be considered as the infeasible strategy. One can interpret this algorithm as a modification of the active-set method so that a weak penetration of constraints is allowed. The “zig-zag” phenomena is suppressed again. Another benefit consists in the fact that the Coulomb friction problem may be solved directly without necessity to introduce the auxiliary problem with Tresca friction. We compare all these algorithms.

Key words

Contact problem, active set method, interior point method, semi-smooth Newton method.

References

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Law of Large Numbers in L_2

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Abstract

Law of Large Numbers is a basic and central phenomenon in both theoretical and practical statistics. It begins any investigation on behaviour of sums of random variables. Considering the sum in L_2 -background, we can receive substantial information for further research. Advantage we emphasize is that in L_2 , it is easy to handle with dependency structure since only covariances come into account. We will present a simple discussion of the topic with an application to weak consistency of statistical estimators.

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Computing Non-Unique Solutions of Quasi-Static Contact Problems with Coulomb Friction

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Abstract

The talk will deal with numerical realization of two-dimensional quasi-static contact problems with Coulomb friction in finite deformations. Using standard numerical methods (e.g. the Newton method, the method of successive approximations), one is able to obtain some solution of these problems without any further information on existence of other solutions. One may face even situations when the standard solvers are not capable of finding any solution at all. This is why more sophisticated techniques have to be explored.

In our contribution we shall follow [1] and [2], where a continuation of the predictor-corrector type was adapted to static contact problems with Coulomb friction and tested on examples with a very small number of degrees of freedom in the framework of small deformations. More precisely, we shall formulate each discretized incremental problem, which corresponds to a static problem, as a system of non-smooth equations involving projections. Consequently, we shall use the cited path-following technique to compute its solutions. As we shall show on a problem coming from technical practice, this approach proves to be able to recover examples of bifurcations in the present framework, as well.

Key words

Contact problem, Coulomb friction, finite deformation, piecewise smooth continuation method.

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Consider Brouwer

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Abstract

L. E. J. Brouwer was well-known Dutch mathematician and philosopher who lived from 1881 to 1966. In classical mathematics, he founded modern topology by establishing the topological invariance of dimension and the famous Brouwer fixed point theorem and he also gave the first correct definition of dimension. Among many fixed point theorems, Brouwer's is particularly significant, due to its use across numerous fields of mathematics. In its original field, this result is one of the key theorems characterizing the topology of Euclidean spaces.

There are two key events in Brouwer's life. The first was a fundamental conflict with David Hilbert in years 1928–1929, after which Brouwer was removed from the editorial board of *Mathematische Annalen*, leading mathematical journal at that time. In August 1929 Brouwer's briefcase was theft in the tram in Brussels and with it his mathematical notebook. Neither the police nor a private detective hired for the purpose were able to find it again. These events caused the shift of his main interest from mathematics to philosophy.

In philosophy, he is known as the founder of the mathematical philosophy of intuitionism and as an opponent to the then-prevailing formalism of David Hilbert. Intuitionism is essentially a philosophy of the foundations of mathematics and views mathematics as a free activity of exact thinking, independent of any language or abstract realm of objects, and therefore bases mathematics on a philosophy of mind. It leads to a form of constructive mathematics, in which some parts of classical mathematics are rejected.

Our paper is focused on Brouwer's life and some of his interesting thoughts from the philosophy of mathematics.

Key words

L. E. J. Brouwer, intuicionism, foundations of mathematics.

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Linear Beams on Nonlinear Foundation

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Abstract

This conference paper follows the articles [1] and [2] and is going to deal with a bending of a beam resting on an elastic foundation. The classical works were concerned with beams in fixed connection with a foundation. Such problems are linear and their solution makes nowadays no fundamental difficulties. However, some applications have the different matter – the beam is not firmly connected with the given foundation. This so-called unilateral case represents a nonlinear problem and cannot be solved by easy means. Regarding beam models, we want to present here the linear ones, i.e. the Euler–Bernoulli beam model, which is suitable for long thin beams, and the Timoshenko beam model that is used for short thick beams.

The usual method for a representation of our problem consists in an extension of the beam equation with a term representing the influence of the foundation, often called the response function. Our new formulation is based upon the idea of problem decomposition. We introduce a supplementary variable linked to the original variable through a simple equality. To handle this constraint we shall utilize a Lagrange multiplier and transfer our problem to a saddle point problem. This way we obtain a mixed formulation. Finally we want to present some solution methods based on the finite element discretization.

Key words

Euler–Bernoulli beam, Timoshenko beam, unilateral foundation, mixed formulation.

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Thanks

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Information Measure for Vague Symbols

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Abstract

The contribution deals with a suggestion of an information measure adequate to fuzzy information sources and their vague symbols and words.

The already existing measures of uncertainty regarding vagueness are focused on entire fuzzy sets, and their main concept is the fuzzy entropy interpreted as characteristics of “distance” between a fuzzy set and its crisp counterpart. These definitions of fuzzy entropy are closely analogous to the concept of probabilistic entropy, as defined by Shannon and Weaver in the classical information theory. The fuzzy set theoretical analogy of the concept of information transmitted by a single symbol of the alphabet is not explicitly considered in the models of fuzzy entropy, however implicitly it is the main component of them.

The close similarity between probabilistic and fuzzy concepts of entropy means that also the concealed implicit model of fuzzy information possesses many properties usual in the probabilistic structures but relatively exotic for the fuzzy sets. Namely, the monotonicity of the fuzzy set theoretical concepts is consequently substituted by the typically probabilistic additivity of principal operations.

The aim of the contributed paper is to suggest new, consequently monotone measure of the fuzzy information for single symbols of the source alphabet, to derive its basic properties, and to study the compatibility between fuzzy set theoretical and probabilistic information measure. Regarding the formal tools used in the definition of fuzzy information, the operation of maximum and minimum form the basic components of the model, which method implies the desired monotonicity of the fuzzy information model.

The main ideas of the contribution are related to the following works.

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Copulas and Their Applications in Multicriteria Decision Support

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Abstract

Copulas were introduced in 1959 by A. Sklar [3] as a tool for capturing the dependence structure of multivariate random variables. Copulas are, in fact, restrictions of multivariate distribution functions with marginals uniformly distributed over $[0, 1]$ to their support, i.e., to $[0, 1]^n$. For more details we recommend the monograph [2].

The importance of copulas is expressed in the Sklar theorem, where the relation $F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n))$ shows the link between the joint distribution function F and marginal distribution functions F_i , connected by a copula C .

Several important classes of copulas, especially 2-dimensional, will be introduced, covering, among others, Archimedean copulas, copulas stable under univariate conditioning, special singular copulas, etc.

In multicriteria decision support, the extension of boolean utility functions to graded utility functions is an important tool enabling to recommend suitable alternatives for further consideration. Boolean utility functions can be seen also as capacities assigning weights to the groups of criteria. One such extension method based on copulas will be presented. Copulas can be seen here as a link between weights and score. In the case of independence copula Π , the Choquet integral is recovered. The comonotone dependence copula *Min* leads to the Sugeno integral.

For symmetric capacities (weights depend on the cardinality of considered groups of criteria only), Π copula yields *OWA* operators, while *Min* yields *OW-Max* operators. For a general copula C , a new class of *OMA* utility functions [1] (Ordered Modular Averages) will be introduced and discussed.

Key words

Copula, multicriteria decision support, utility function.

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On Criterion Robust Designs for Experiments with Blocks of Size Two

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Abstract

Designs with block of size two can be applied in diverse situations, e.g., for two-color microarray experiments (see [1]). The simplest underlying mathematical model is that of a two-way ANOVA without interactions, with block (microarray) parameters β_1, \dots, β_b and treatment parameters τ_1, \dots, τ_t under the requirement that each block contains exactly two treatments.

Formally, the measurements Y_k can be expressed by $Y_k = \beta_{i_k} + \tau_{j_k} + \varepsilon_k$, $k = 1, \dots, 2b$, where $j_1, \dots, j_{2b} \in \{1, \dots, t\}$, $j_{2l-1} \neq j_{2l}$, $i_{2l-1} = i_{2l} = l$ for $l = 1, \dots, b$ and ε_k are i.i.d., $E(\varepsilon_k) = 0$. The design of such experiment corresponds to a selection of the treatments j_1, \dots, j_{2b} and can be represented by its concurrence graph with t vertices and b edges joining vertices j_{2l-1} and j_{2l} for $l = 1, \dots, b$.

The aim of designing of an experiment is to choose one of the possible designs to gain as much information about τ_1, \dots, τ_t as possible, according to a real-valued optimality criterion. However, as pointed out in [1], the structure of the designs depends on the optimality criterion used. Hence, a “criterion-robust” design, i.e., a design efficient with respect variety of criteria, would be valuable.

Based on the results in [2], we will show that for the comparison of criterion robustness of designs, the key is the criterion of E -optimality that maximizes the second smallest eigenvalue λ_2 of the normalized information matrix $M(\xi)$ of the design ξ , or, equivalently, the algebraic connectivity of the corresponding concurrence graph. In particular, the value $\text{eff}(\xi|E) = 2\lambda_2(M(\xi))/t$ gives a lower bound for the efficiency of a design ξ with respect to a very large class of the so-called orthogonally invariant criteria. For different values of b and t , we will exhibit the criterion robust designs represented by their concurrence graphs, together with the lower bound for their efficiency.

Key words

Block designs, microarray experiments, E -optimality.

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Physical Heights Determination by Satellite and Gravity Measurements

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Abstract

Physical height is a parameter defining vertical information between two points. Physical height is characterized by geometric and physical aspects. The determination of physical heights between points includes both of these aspects.

The classical determination of physical heights uses geometrical levelling and gravity measurements between points of levelling network.

The modern determination of physical heights combines of space geodesy and gravity measurements. The relationship for determination of geopotential differences between two points can be expressed by formula

$$\Delta W_{o,p} = W_o - W_p, \quad (1)$$

where W_o is mean value of the geopotential of the mean sea surface at the time period of satellite altimeter measurements and

$$W_p = W_p^{GGM} + \delta W_p, \quad (2)$$

where W_p^{GGM} is a global geopotential part determined using global gravity field model (for ex. EGM2008) and δW_p is correction of global geopotential value computed from gravity measurements (gravity disturbance) in the area around point P by applying a modified second geodetic boundary value problem. The theoretical principle of such a solution and its practical application in the area of Slovakia will be presented.

Key words

Physical heights, global gravity model, GNSS positioning, gravity disturbance.

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Equidistant General Exponential Analysis with Some Known Exponents

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Abstract

In this paper we proof the formula, which can be useful in the analysis of certain processes, namely processes of the type

$$f(x) = \sum_{i=1}^r \sum_{j=1}^{k_i} a_{ij} x^{j-1} e^{\lambda_i x} + \sum_{i=1}^s \sum_{j=1}^{l_i} x^{j-1} e^{\alpha_i x} (b_{ij} \cos \beta_i x + c_{ij} \sin \beta_i x)$$

with some of parameters $\lambda_1, \dots, \lambda_r, \alpha_1, \dots, \alpha_s, \beta_1, \dots, \beta_s$ known. Function $f(x)$ is called E -function of type

$$n(r; s; \lambda_1(k_1), \dots, \lambda_r(k_r), \alpha_1, \dots, \alpha_s, \beta_1(l_1), \dots, \beta_s(l_s))$$

or E -function of type $n(r; s)$ or E -function of type n . By using this formula, the analysis of E -function of type $n(r; s)$ can be transfered to solving of the system of linear equations, finding roots of polynomial and computing logarithms of real numbers.

The analysis of E -function of type $n(r; s)$ is useful in many applications, for example in regression analysis, creep of concrete, disease diagnosis, in electrical phenomenon describing, in modeling relationships in economics.

Key words

E -function, relationships between roots and coefficients of polynomial, equidistant distance of points.

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Applications of Bivariable Functions on Quantum Logics

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Abstract

A quantum logic is an orthomodular lattice with at least one state [1]. In our contribution we concern to bivariable functions on quantum logics. Under special marginal conditions these functions lead to s-maps and d-maps, which on a Boolean algebra represent measures of intersection and measures of symmetric difference of two elements, respectively. A special bivariable maps on a quantum logic were introduced in [2,3], and studied for example in [4]. d-maps have several applications in a quantum logic theory:

- The properties of 2-dimensional d-map induced by 2-dimensional s-map play crucial role for the existence of a 3-dimensional s-maps.
- It is possible to characterize a centrum in various types of quantum logics via special bivariable functions.
- d-maps allow to study various types “undistinguishable” elements on a quantum logic.

Key words

Quantum logic, state, bivariable function, symmetric difference, Boolean algebra.

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Classification of Association Coefficients and Its Application in Cluster Analysis

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Abstract

Creating groups with similar characteristics can be observed in all directions and areas of organic and anorganic nature, social relations, medicine etc. Clustering is a process of finding groups of objects with similar properties. It is a very important task in data mining. The basic problem is to determine the optimal cluster amount because the number of classes is not known before the process finalization. To achieve correct results in clustering, researchers use various clustering methods for the same objects. These results evoke the question of accuracy of the constructed clusters. Do the objects inside the clusters have the greatest degree of similarity? For comparison of created clusters many methods have been appointed. One of them, which appears to be very useful, is the association coefficients's metod. The aim of this work is the classification of the most commonly used association coefficients in engineering praktice. Their application to assess the results of different methods of clustering is also investigated. All these coefficients was divided into three groups. The coefficients were expressed in terms of the operator of symmetric diference. Moreover, they were expressed also by a particular rational function. In this paper the hydrological data, which are processed by different methods of clustering, are used.

Key words

Similarity, dissimilarity, symmetric diference, association coefficients, rational function.

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Several Definitions of Probability on Systems of Fuzzy Sets

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Abstract

The classical probability theory (by Kolmogorov) is based on the notion of σ -algebra. Its algebraic generalization—a *Boolean σ -algebra*—need not be represented by a collection of sets.

Among extensions to fuzzy logic, a successful approach is based on σ -complete MV-algebras [2]. The development of probability theory has brought some difficulties which could be overcome by the use of σ -complete MV-algebras with product [5], where the product is introduced in a unique way if the σ -complete MV-algebra is representable by a collection of fuzzy sets.

Conjunction and disjunction in a fuzzy logic can be interpreted by triangular norms and conorms different from the Łukasiewicz operations used in MV-algebras. This approach is based on the notion of a *tribe* of fuzzy sets as a natural generalization of a σ -algebra [1].

A new algebraic approach has been suggested in [3,4]. It defines a *fuzzy σ -algebra* as a common generalization of the above notions. Still a reasonable measure and probability theory can be developed in this context.

Key words

Probability, fuzzy set, clan, tribe, MV-algebra.

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Rank Tests of Symmetry and R-Estimation of Location Parameter under Measurement Errors

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Abstract

This contribution deals with the hypotheses of symmetry of distributions with respect to a location parameter when the response variables are subject to measurement errors. Rank tests of hypotheses about the location parameter and the related R-estimators are studied in an asymptotic set up. It is shown, when and under what conditions, these rank tests and R-estimators can be used effectively, and indicate the effect of measurement errors on the power of the test and on the efficiency of the R-estimators. Some simulation results are provided to illustrate the influence of measurement errors on the power and the accuracy of the estimators.

Key words

Location, measurement error, R-estimate, rank, rank test.

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Higher-Order Fuzzy Logic

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Abstract

Mathematical fuzzy logic is a well established formal theory enabling to develop models of human reasoning affected by the vagueness phenomenon. Besides several kinds of propositional and first-order calculi, also higher-order fuzzy logic was developed which is, in analogy with classical logic, called *fuzzy type theory* (FTT) (cf. [1]). Truth values in FTT form an ordered structure with more specific properties. The fundamental class of the latter is formed by *MTL-algebras* which are prelinear *residuated lattices* $\mathcal{L} = \langle L, \vee, \wedge, \otimes, \rightarrow, \mathbf{0}, \mathbf{1} \rangle$ in which \rightarrow is tied with \otimes by the adjunction. The first kind of FTT has been developed for IMTL_{Δ} -algebra of truth values in [2]. The latter is the MTL-algebra keeping the law of double negation and extended by a unary Δ -operation (in case of linearly ordered \mathcal{L} , Δ keeps $\mathbf{1}$ and assigns $\mathbf{0}$ to all the other truth values). The most distinguished algebra of truth values for FTT is the standard Łukasiewicz $_{\Delta}$ MV-algebra $\mathcal{L} = \langle [0, 1], \vee, \wedge, \otimes, \rightarrow, 0, 1, \Delta \rangle$. Another general class of algebras motivated by FTT is formed by EQ-algebras $\mathcal{E} = \langle E, \wedge, \otimes, \sim, \mathbf{1} \rangle$ where \wedge is meet, \otimes is a monoidal operation (possibly non-commutative), and \sim is a fuzzy equality (equivalence). Implication in EQ-algebras is a derived operation $a \rightarrow b = (a \wedge b) \sim a$ which is not tied with \otimes . The corresponding EQ-FTT has been developed in [3]. Syntax of FTT is a generalization of the lambda-calculus constructed in a classical way, but differing from the classical one by definition of additional special connectives, and by logical axioms. The fundamental connective in FTT is *fuzzy equality* \equiv , interpreted in truth values by fuzzy equivalence and otherwise by a reflexive, symmetric and \otimes -transitive binary fuzzy relation. Generalized completeness theorem holds for all kinds of FTT. We will also mention few results in applications of FTT to formal theory of human reasoning which encompasses mathematical models of the meaning of special natural language expressions.

Key words

Mathematical fuzzy logic, algebra of truth values, fuzzy type theory.

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Fuzzy Extension of the Weighted Average Operation and Defuzzification of the Resulting Fuzzy Number

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Abstract

The contribution deals with the fuzzy extension of the weighted average operation. First, the convenient ways how uncertain weights and weighted values can be modeled by fuzzy vectors will be briefly introduced. It will be shown that, in comparison to a tuple of fuzzy numbers that have been used for modeling uncertain values of particular weights and weighted values up to now, fuzzy vectors extend the possibilities of utilizing the vague expert information concerning the weights and weighted values. Next, we focus on computation of a fuzzy weighted average of a fuzzy vector of weighted values with a fuzzy vector of weights. A general formula will be derived and its special forms will be studied. The advantage of the approach presented in the contribution is that the resulting fuzzy weighted average is not overly imprecise since all available information about its variables is involved in computation. On the contrary, an obvious disadvantage is the possible increase of computational complexity. This fact will be illustrated by several examples. Finally, the problem of defuzzification of the resulting fuzzy weighted average will be briefly discussed. Some possible ways of defuzzification will be mentioned and it will be studied how the various interactions among the weights and among the weighted values affect the process of defuzzification. It will be shown that, in comparison with other methods, the expected value of the fuzzy weighted average reflects also the interactions among the weights and among the weighted values that do not affect its membership function.

Key words

Fuzzy vectors, extension principle, fuzzy weighted average, expected value of a fuzzy number.

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Strange Design Points in Linear Regression

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Abstract

We discuss, partly on simple examples, several intuitively unexpected results in a standard linear regression model. First, for completeness we repeat briefly known conditions under which a design point is sensitive to the presence of outliers. Then we demonstrate that direct observations of the regression curve at a given point can not be substituted by observations at several very close neighbourhood points. On the opposite, we show, with the help of the Elfving theorem, that substitution by observations at several distant design points may improve the variance of the estimator. Finally, in an experiment with correlated observations we show somehow unexpected necessary and sufficient conditions under which a design point gives no or very little information about the estimated parameters, and so can be excluded from the design.

Key words

Singular models, optimal design, correlated observations.

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F-Transform—A New Paradigm in Fuzzy Modeling

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Abstract

The theory of F-transform (short name for fuzzy transform) is a modern theoretical tool for fuzzy modeling. On the basis of this theory, a methodology with many applications in the areas of data analysis, image processing, time series analysis and forecasting has been developed (see [1]). Due to clear theoretical basis and common effort of many researches, both theory and applications have been extensively developed in recent years.

The theory of F-transform has a comprehensive mathematical background which is comparable with the well-known transforms such as Laplace, Fourier, and wavelet ones. From this point of view, the F-transform brings fuzzy modeling to a level of pure mathematical modeling. Its results are comparable and in some respects surpass the classical models, e.g. in time series analysis and forecasting, or in image processing (see [2]).

In [3], we showed that the F-transform can be generalized to the case F^m ($m \geq 0$), where the F^m -transform components are polynomials of degree m . We proved that every polynomial component approximates a certain restriction of an original function and that as the degree of the polynomial increases, so does the quality of approximation. We provided a detailed characterization of the F^1 -transform with linear polynomials as components. Finally, we introduced an inverse F^m -transform and showed that it approximates the original function on the whole domain. Moreover, we discussed the quality of approximation in terms of the inverse F^m -transform in two approximation spaces (specifically, the space of continuous functions and the space L_1), where we proved a uniform convergence of a sequence of inverse F^m -transforms.

Key words

Fuzzy transform, mathematical modeling.

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Thanks

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Statistical Analysis of High-Dimensional Data in Cancer Research

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Abstract

The discovery of small non-coding RNAs and the subsequent analysis of microRNA expression patterns in human cancer specimens have provided completely new insights into cancer biology. MicroRNA based microarrays are being increasingly used in cancer research for diagnosis and classification, prediction of prognoses and better understanding of molecular variations among tumours or other biological conditions. They allow us to measure hundreds of transcripts simultaneously in one single experiment. Since the dimension of the feature space (the number of microRNAs) is much greater than the number of tissues, the problem in statistical analyses of these data sets becomes non-standard and represents a challenge for both statisticians and biologists. The marker selection among hundreds of microRNAs can improve cancer diagnostic and drug response prediction. This can be used for development of microRNA expression based diagnostic tests to guide therapy in cancer patients.

The main goal is to identify differentially expressed microRNAs associated with experimental conditions (drug response). Application of linear statistical model allows additional covariates for complex experimental design. To test statistical significance, the empirical Bayes moderated statistics is used.

Moreover, we focus on classification and prediction of a sample based on microRNA expression profiles. We apply machine learning approach to perform microRNA selection based on random forests.

Key words

Microarray, microRNA, linear model, empirical Bayes, machine learning.

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Interval and Fuzzy Linear Programming (Demonstrated by Example)

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Abstract

In this paper, the well known problem of linear programming (LP) with interval (ILP) and fuzzy coefficients (FLP) is investigated. Pessimistic and optimistic solution of ILP problem is defined and discussed. Then FLP problem with fuzzy inequality relations, fuzzy maximization and fuzzy constraints is dealt with. Fuzzy coefficients are taken in the form of fuzzy intervals, particularly triangular fuzzy numbers. Arithmetic operations with fuzzy numbers are introduced and 2 types of inequality relations between fuzzy numbers, particularly pessimistic and optimistic-inequality relations are investigated. Then α -feasible solution of FLP problem and α -optimal solution of FLP problem is considered. Other approaches are also mentioned and advantages/disadvantages of various approaches are discussed. The introduced concepts are demonstrated on simple examples solved by Excel-Solver and displayed by a graphical aid.

Key words

Interval linear programming, fuzzy linear programming, fuzzy coefficients, fuzzy inequality relations, fuzzy maximization, fuzzy constraints, α -feasible solution, α -optimal solution.

Exploratory Projection Pursuit: A Review and New Proposals

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Abstract

Exploratory Projection Pursuit is defined and studied in the pioneering papers by Friedman and Tukey (1974), Huber (1985), Friedman (1987) and Jones and Sibson (1987). It consists in looking for “interesting” projections of high dimensional data on one or two dimensional subspaces. The interestingness of a projection is measured by a projection index that has to be maximized over all the possible projection subspaces. In the years that followed the founder papers, several proposals have been made on the subject. However, the methodology is not implemented in well-known statistical softwares and there are very few recent references. In this presentation, we give a survey on Exploratory Projection Pursuit from the origin until nowadays and make new proposals. The proposals include new projection indices dedicated to the detection of outliers and clusters and a new computational strategy based on the analysis of several runs of local optimization algorithms (see also Ruiz-Gazen et al., 2010, and Berro et al., 2011).

Key words

Cluster, optimization algorithm, outlier, projection index

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Disaster Decision Making Support A Linguistic Fuzzy Model for the Emergency Medical Rescue Services

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Abstract

The decision making process of the Emergency medical rescue services operations centre during disasters involves a significant amount of uncertainty. Our goal is to speed up the decision making process and to eliminate mistakes—particularly in the case of disasters resulting in large number of injured people. A multiphase linguistic fuzzy model is introduced to assist the operator during the initial phase of the medical disaster response. The model is able to deal with uncertain input data, it can estimate the severity of the disaster, the number of injured people, and the amount of forces and resources needed to successfully deal with the situation. The need of reinforcements is also considered. Fuzzy numbers, linguistic variables and fuzzy rule bases are applied to deal with the uncertainty. The concept of Fuzzy number α -degree upper bound is introduced. This concept allows us to assess the degree of satisfaction of fuzzy constraints. Results derived by the model are available both as fuzzy sets and linguistic terms. The proposed model respects the uncertainty of information that accompanies emergency calls and is designed to derive appropriate decision support data. Results provided by the model can prove useful for emergency planning and disaster management in the Czech Republic as well.

Key words

Linguistic fuzzy modeling, decision making support, linguistic scale, emergency medical rescue services.

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A Note on Design Optimization of a Beam: Algebraic Sensitivity Analysis

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Abstract

Shape design optimization problems have been subjects of considerable research and are of concern in many engineering applications. In their practical realization one may meet some difficulties and we should pay attention to a thorough mathematical analysis in order to obtain additional information on the problem that can be useful in computations.

In general a discrete design optimization problem leads to a nonlinear programming problem where the objective function can be nonsmooth and non-differentiable. Moreover, every evaluation of the objective function involves a solving of a state problem. State problems are usually represented by a linear (nonlinear) system of algebraic equations or a nonlinear programming problem. Consequently, the outer optimization algorithm should use as few function evaluations as possible. Thus some gradient information is needed. Since the objective function is in fact a composite mapping, the task to obtain a gradient can not be so straightforward. Sensitivity analysis in shape optimization deals with computations of derivatives of solutions of the state problems and cost functionals with regard to design variables.

In this note we deal with algebraic form of design optimization of an elastic beam with several variants of a state problem. A beam with bilateral elastic foundation, a beam with rigid obstacle and a beam with unilateral elastic foundation are considered. An adjoint state technique that help us obtain the gradient (subgradient) of the objective function efficiently is described.

Key words

Shape design optimization, beam problem, sensitivity analysis.

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Mathematical Models of Academic Staff Performance Evaluation

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Abstract

In the paper we describe the development process of the academic staff performance evaluation model for Palacký University in Olomouc (Czech Republic). Various alternatives of the mathematical solution are discussed here. All the models share the same basic idea—we evaluate the staff member's performance in the area of Pedagogical Activities and in the area of Research and Development. The input data for the models are obtained from structured forms containing information about all the activities performed by a current staff member in the respective year. We require an aggregated piece of information concerning the yearly performance of a particular staff member at a current work position (achievement of standard performance, achievement of excellence etc.). In the first part of the paper we analyse a group of models that share the algorithm for normalized partial evaluations determination in both areas of interest (Pedagogical Activities, Research and Development); the partial evaluation normalization function is determined by the scores for standard and excellent performance (defined by the evaluator for different work positions and for both areas of interest separately). Models within this group differ by the aggregation operator used to calculate the overall performance evaluation—weighted arithmetic average (WA), OWA and WOWA. The second part of the paper is devoted to the description of a model where partial evaluations are determined simply as multiples of standard score for the current work positions and area of interest, but the aggregation of these partial evaluations is performed by a fuzzy rule based system. This fuzzy model is currently being implemented at Palacký University, Faculty of Science.

Key words

Evaluation, academic staff, aggregation, fuzzy model.

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The Application of Linguistic Fuzzy Models to the Analysis of Adolf Loos' Villas

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Abstract

The fuzzy set theory makes it possible to process mathematically uncertain information (uncertain values, uncertain relations). Similarly a natural language works fluently with uncertain notions. In the theory of architecture, the common rules characterizing works of a famous architect are often described verbally in literature. If the verbal description is formalized enough, then its meaning can be modeled mathematically by means of fuzzy sets, and the vague knowledge concerning the architect can be stored in a computer. Linguistic fuzzy model that is used in this paper to model the vague common rules of the architectural work of Adolf Loos illustrates the possibility of such approach.

Adolf Loos ranks among the most important architects of the first half of the 20th century; the most important are his projects of villas. When designing the villas, Adolf Loos applied his Raumplan (i.e. Space Plan) principle. The principle said that no floor plans, no vertical sections and no facades should be designed. According to the principle, particular spaces whose sizes and highs were determined by their functions should be designed and located on different height levels in the house.

In the paper, a fuzzy model containing rules typical for the design of the Adolf Loos' villas will be presented. The fuzzy model consists of several mutually interconnected fuzzy rule bases and it is created in MATLAB. The basic structure of the model (the relationship among particular rule bases, the choice of input and output variables in each of the rule bases) was created by means of the ordinary sequence of steps in the design process. The structure of the fuzzy models reflects also the Raumplan principle. The particular rules of any fuzzy rule base are derived from the Loos' villas; the designed villas represent fuzzy data for forming the rule bases.

Key words

Fuzzy models, expert systems, architecture.

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Estimation of Social Inclusion Indicators with Bounded Influence of Incomes by Tail Modeling

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Abstract

The estimation of poverty and social inclusion is one of the most crucial topics in the EU, and deals as basis for decisions about future policy in the EU.

However, most of these indicators are highly influenced by outliers in the upper tail of the income distribution and standard estimations of the indicators leads to wrong conclusions for policy makers.

This presentation investigates the use of robust Pareto tail modeling to gain bounded influence of outlying observations to the indicators obtained from complex survey designs. Above at an estimated threshold, values are replaced by draws from a Pareto distribution. The crucial point is to estimate the shape parameter of the Pareto distribution, which is done by robust methods. In addition, sampling weights are considered through the estimation procedure.

For the corresponding simulation study the generation of close-to-reality population data are necessary for which samples are drawn repeatedly using complex sampling designs. Outliers and missing values have then to be introduced in a realistic manner to simulate peoples behaviour when responding to the corresponding questionnaire.

A small outline of the large-scale simulation study is shown as well as the mathematical concept of the tail modeling approach together with an application of the implemented tools ([1]).

Key words

Poverty measurement, pareto tail modeling, robust methods, complex simulations.

References

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Robust Multivariate Analysis in R

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Abstract

The estimates of the multivariate location vector $\boldsymbol{\mu}$ and the scatter matrix $\boldsymbol{\Sigma}$ form the input to many classical multivariate methods. The most common estimators of the multivariate location and scatter are the sample mean $\bar{\boldsymbol{x}}$ and the sample covariance matrix \boldsymbol{S} , i.e. the corresponding MLE estimates. These estimates are optimal if the data come from a multivariate normal distribution but are extremely sensitive to the presence of even a few outliers (atypical values, anomalous observations, gross errors) in the data. If outliers are present in the input data they will influence the estimates $\bar{\boldsymbol{x}}$ and \boldsymbol{S} and further will worsen the performance of the classical multivariate procedure based on these estimates. Therefore it is important to consider robust alternatives to these estimators and actually in the last several decades a lot of effort was devoted to development of affine equivariant estimators which have also high breakdown point [1].

The routine use of robust methods in a wide area of application domains is unthinkable without the computational power of today's personal computers and the availability of ready to use implementations of the algorithms. A unified computational platform organized as common patterns which we call statistical design patterns in analogy to the design patterns widely used in software engineering is presented and its implementation in R is discussed [2, 3].

Some of the most popular robust estimation methods are presented and their performance is evaluated and compared by simulation in a variety of situations. Substituting the classical location and scatter estimates by their robust analogues is the most straightforward method for robustifying many multivariate procedures like principal components, discriminant analysis and others. The reliable identification of multivariate outliers which is an important task by itself, when performed by means of robust estimators, is another approach to robustifying many classical multivariate methods.

Key words

Robust estimation, multivariate analysis, MCD, R.

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Delimitation of Forest Health Zones Using Multivariate Statistical Analysis

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Abstract

Delimitation of zones of forest health hazard rate is reflecting ongoing changes in vegetative environments. Zones of forest health hazard rate serve as a foundation of methodology of effective forest management. Current system of zonation in Czech Republic is unsatisfactory, because no regard is taken of causes of wither of forest ecosystems. In this paper, we present new approach based on multivariate statistics techniques, followed by constructing model based on fuzzy set theory. Multivariate statistical techniques like cluster analysis and discriminant analysis are used to identify the factors, which have significant effect on the forest health hazard. Groups of factors are selected, categorized, and model for hazard rate assessment is constructed using fuzzy sets. Results in a form of maps and conceptual scheme of model are presented and discussed.

Key words

Zones of forest health, cluster analysis, discriminant analysis.

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Statistical Processing and Design of Experiment of ϵ -Fe₂O₃ Hysteresis Loops

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Abstract

Analyzing the profile of a hysteresis loop of a (nano)material, measured at a given temperature and under varying external magnetic field, brings valuable information on its magnetic properties. Deriving the values of hysteresis parameters enables decision if the (nano)material meets material requirements of a given application. This is a case of ϵ -Fe₂O₃ which has been recently found as a perspective candidate for high-coercivity applications. Generally, the hysteresis loop can be described within the Langevin approach employing the Langevin function given by

$$y = \theta_1 \cdot \coth [\theta_2 \cdot (x - \theta_3)] - \frac{\theta_1}{[\theta_2 \cdot (x - \theta_3)]},$$

where y denotes the magnetization of a material, $\Theta = (\theta_1, \theta_2, \theta_3)'$ stands for an unknown vector parameter including magnetic and hysteresis parameters of a material and x is the induction of an external magnetic field.

In this work, we report on estimation of Θ and fitting the hysteresis loops of ϵ -Fe₂O₃ nanoparticle systems. In order to meet this objective, a linearized regression model has been constructed enabling to derive ϵ -Fe₂O₃ hysteresis parameters and consequently the area of a respective ϵ -Fe₂O₃ hysteresis loop.

Key words

Magnetization, hysteresis loops, Langevin function, nonlinear and linearized regression models, design of experiment.

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Concept of Data Depth and Depth Based Classification

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Abstract

Data depth is very useful tool for nonparametric multivariate data inference. Basic tools such as descriptive characteristics of location, scale, skewness and kurtosis of multivariate distribution can be based on data depth. These characteristics were proposed by an outstanding article by Liu, Parelius and Singh in 1999 [3]. The paper deals also with visualizing these features via one-dimensional curves.

More sophisticated methods based on data depth are reviewed in the article [4] by Serfling. Depth-based statistical procedures include tests of multivariate symmetry, diagnosis of nonnormality, comparison of several distributions (e.g. tests of equal scales), outlier identification, statistical process control procedures, multivariate density estimation and some others.

During the last ten years quite a lot of effort has been put into development of an alternative nonparametric approach, which uses methodology of data depth for solving the classification problem. This classical problem consists in creating a rule for distinguishing objects of several groups. Several classifiers based on data depth have been proposed. Unfortunately, many of them are useful only under quite restrictive assumptions. Current research aspire to broaden the class of problems that can be solved by methods based on data depth.

Key words

Data depth, nonparametric, classification.

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Fuzzy Information in Statistics

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Abstract

In standard statistics data are considered to be numbers, vectors, or classical functions. This is assumed in classical statistics as well as in Bayesian inference.

But measurement results from continuous variables are not precise real numbers, but more or less non-precise. The best up to date mathematical description of measurement data is by so-called fuzzy numbers, fuzzy vectors, or fuzzy valued functions respectively. Fuzzy numbers are generalizations of real numbers and intervals. For example environmental data are frequently connected with remarkable imprecision which makes it necessary to model this fuzziness in order to obtain realistic results from statistical analyses. The generalization of statistical methods to the situation of fuzzy data is possible and will be explained in the talk.

Another kind of fuzzy information is a-priori information in Bayesian inference. Classical a-priori distributions are a critical topic in Bayesian statistics. The more general concept of so-called fuzzy probability distributions is a suitable concept to describe fuzzy a-priori information. A generalization of Bayes' theorem for fuzzy a-priori distributions and fuzzy data which keeps the sequential nature of the updating procedure of Bayes' theorem will be presented. Moreover generalizations of the concepts Bayesian confidence regions and predictive distributions will be given.

Key words

Fuzzy Bayesian inference, fuzzy data, fuzzy information, fuzzy models, fuzzy probability distributions, statistical inference.

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Principles of Collective Decisions

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Abstract

This talk offers a brief introduction to some of the fundamental issues and results of theory of collective decision-making in the situations wherein a group of individuals with diverse preferences is collectively trying to choose fairly among a finite number of mutually exclusive alternatives.

An obvious way in which a group of individuals can aggregate preferences of its members into group preferences is to use a voting procedure. However, it has been known for a long time that results of voting procedures may be in contradiction with some basic requirements of rationality when the group has two or more members and there are at least three alternatives.

We begin with simple examples to introduce some of the desirable properties of aggregation procedures and to explain difficulties in satisfying all of these properties simultaneously by a single procedure. Then we define three general concepts of aggregation: a collective choice rule, a social welfare function, and a social decision function; and three basic requirements: unrestricted domain, weak Pareto efficiency, and individual liberty. We conclude with discussing results on the possibility and impossibility of satisfying the requirements by aggregation procedures. In particular, we present three basic results in this line of research: Arrow's impossibility theorem, the Gibbard-Satterthwaite manipulability theorem, and Sen's liberal paradox.

Key words

Social choice, social welfare function, social decision function, manipulability, liberal paradox.

Thanks

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On Testing Problems in Linear Models with Variance-Covariance Components

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Abstract

Linear models with variance-covariance components are used in a wide variety of applications. In some situations it is possible to partition the response vector into independent subvectors, as in longitudinal models where the response is observed repeatedly on a set of sampling units (see e.g., Laird & Ware 1982), but in many cases such partitioning is not straightforward, e.g., in some geodesic or geophysical applications (see e.g., Kubáček, Kubáčková, Volaufová 1995). Often the objective of inference is testing linear hypotheses about the mean of the response. How to do that is a question that has kept many statisticians busy for several decades. Even assuming multivariate normality, it is not clear what test to recommend except in a few special settings, such as balanced or orthogonal designs. Here we shall investigate some statistical properties of alternative test procedures, such as accuracy of p -values and powers of approximate (Vonesh & Carter 1987, Kenward & Roger 1997) and exact (Crainiceanu & Ruppert 2004) tests.

Key words

Fixed effects, approximate test, exact test, accuracy of p -value.

Continuation of Professor Kubáček's Research Work: Some Miscellaneous Examples

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Abstract

Professor Kubáček contributed to solution of a tremendous number of real problems. Because of his excellent knowledge about the state of the practice, sometimes he oneself said “This solution will be sufficient for some decades”. Three scientific generations of Professor Kubáček demonstrate on three chosen miscellaneous examples the continuation of Professor Kubáček's research work.

Key words

Calibration problem, digitized measurements, model for longitudinal data.

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Thanks

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Properties and Applications of (max,min)-Linear Equations and Inequalities

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Abstract

As an introduction, motivating examples from the area of transportation problems with both fuzzy and crisp capacities, as well as some fuzzy set covering problems will be presented. It will be shown that the problems lead to solving special systems of equations and inequalities, which are referred to in the literature as (max,min)-linear equations or inequalities. In such systems, so called (max,min)-linear functions occur. If the functions occur on only one side of equations or inequalities and the other side is a constant, we speak about one-sided equations or inequalities, otherwise we speak about two-sided equations or inequalities. The one-sided equations of (max,min)-equations appeared in the fuzzy set literature as relation equations (see e.g. [2]). It can be shown that any such system of equations and inequalities (both one-sided and two-sided) can be transformed to an equivalent system having the following standard form:

$$\max_{j \in J}(\min(a_{ij}, x_j)) = \max_{j \in J}(\min(b_{ij}, x_j)), \quad \forall i \in I, \quad (1)$$

$$x = (x_1, \dots, x_n) \in F^n \subseteq R^n, \quad \underline{x} \leq x \leq \bar{x}, \quad (2)$$

where I, J are finite index sets, F is a closed subset of R^n , $\underline{x}, \bar{x} \in F^n$, $a_{ij}, b_{ij} \in F$ $\forall i \in I, j \in J$ are given elements. In the fuzzy set context we set usually $F = [0, 1]$. Properties of the system (1), (2) will be presented and algorithms for solving the system, as well as some related optimization problems will be proposed. Results from [1], [3] are used.

Key words

(max,min)-linear equation, fuzzy relation equations, fuzzy sets covering, fuzzy capacitated transportation problems.

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Thanks

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The Preconditioning of Linear Systems in Interior Point Method

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Abstract

The paper deals with a precondition of a conjugate gradient method used for the solution of linear systems in a primal-dual interior point method for minimizing strictly quadratic objective functions subject to separable quadratic constraints. This minimization problem has a form

$$\min_{x \in \Omega} \frac{1}{2} x^\top A x - x^\top b \quad (3)$$

with $\Omega = \{x \in \mathbb{R}^{3m} : x_i \geq l_i, x_{i+m}^2 + x_{i+2m}^2 \leq g_i^2, i = 1, \dots, m\}$, where $A \in \mathbb{R}^{3m \times 3m}$ is symmetric, positive definite and $b \in \mathbb{R}^{3m}$. Such minimization problem arises from the discrete dual formulation of contact problems with given friction in three space dimensions (see [1]). The matrix A is a large scale matrix defined as a product $A = BK^{-1}B^\top$, where $K \in \mathbb{R}^{3n \times 3n}$ is stiffness matrix, n is number of nodes in discretization of the primal formulation and $B \in \mathbb{R}^{3m \times 3n}$ is a matrix which projects the displacement at contact nodes to normal and tangential directions. The direct assembling of matrix A is very time-consuming, hence only matrix-vector operations with A may be performed that requires to use an appropriate iterative method. The matrix of the linear system arising from interior point methods is ill conditioned so that a preconditioner is needed. This matrix can be reduced to symmetric but indefinite *augmented* matrix or to positive definite *normal* matrix. We present preconditioners of the augmented matrix based on Schur complement, which belongs to indefinite class of preconditioner (see [2]), and their simplifications in our problem. Further we introduce simple preconditioners for the normal matrix.

Key words

Precondition, conjugate gradient method, interior point method, contact problems.

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Overview of Recent Results in Multivariate Linear Models

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Abstract

The Extended Growth Curve Model (ECGM) is a multivariate linear model connecting different multivariate regression model in sample subgroups through common variance matrix. It has the form:

$$Y = \sum_{i=1}^k X_i B_i Z_i' + e, \quad \text{vec}(e) \sim N_{n \times p}(\mathbf{0}, \Sigma \otimes I_n). \quad (4)$$

Here, matrices X_i contain subgroup division indicators, and Z_i corresponding regressors. If $k = 1$, we speak about (ordinary) Growth Curve Model.

The model has already its age (it dates back to 1964), but it has many important applications. That is why it is still intensively studied. Many articles investigating different aspects or special cases of the model appeared in recent years. We will try to summarize the progress done so far.

Key words

Growth curve model, extended growth curve model, multivariate linear model.

References

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Thanks

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Professor Kubáček celebrating his 80th birthday



Motto:

*Škola buď dílnou lidskosti
a dětská léta ať neplynou nadarmo!
Ať každý učitel pracuje s radostí
a pokud možno i zadarmo!*

Emil Calda: 'J. A. Komenský',
Úvod do obecné teorie prostoru,
Karolinum, Praha, 2003

*May school be a workroom of humanity
And the years of childhood not in futility flee
May each teacher find joy in their activity
And, if possible, do their job for free!*

Emil Calda: 'J. A. Komenský',
An introduction to the general theory of space
Translation: Ivo Müller

Lubomír Kubáček

Professional Curriculum Vitae

Date and place of birth: February 1st, 1931, Bratislava

Brief portrayal: Slovak mathematician, geodesist, and educator

University diplomas:

Slovak Technical University, Bratislava (1954, Ing., geodesy);

Faculty of Science, Komenský University, Bratislava (1964, graduated mathematician, probability theory and mathematical statistics)

Course of employment:

1954–1962 Institute of Geodesy, Bratislava (Head of Computation Department);

1962–1981 Institute of Measurement, Slovak Academy of Science, Bratislava (Head of Mathematics Department);

1981–1994 Mathematical Institute, Slovak Academy of Science, Bratislava (senior scientific worker, Director);

1994–now Faculty of Science, Palacký University, Olomouc

(Head of Department of Mathematical Analysis and Applications of Mathematics, Professor)

Academic degrees:

Ing. (geodesy) Slovak Technical University, Bratislava, 1954;

RNDr. (probability theory and mathematical statistics), Faculty of Science, Komenský University, Bratislava, 1969

Scientific degrees:

CSc. (measuring technology) Institute of Measurement Theory, Slovak Academy of Science (SAS), 1965;

DrSc. (physics-mathematics) Slovak Technical University, 1980;

dr. h. c. Slovak Technical University, 2002;

Honorary Member of Scholarly Society of Slovak Academy of Science, 2005

Scientific-pedagogical degrees:

doc. (probability theory and math. statistics) Komenský University, 1991;

prof. (probability theory and math. statistics) Komenský University, 1991

Corresponding member of Slovak Academy of Science (probability theory and mathematical statistics), 1987

Fields of scientific interest: estimation theory, singularities in linear statistical models, weak non-linearity of statistical models, multivariate linear and weakly non-linear statistical methods, statistical theory of geodetic networks.

Fields of pedagogical interest: fundamentals of probability theory and mathematical statistics, estimation theory, linear statistical models, regression analysis, statistical theory of experiments.

Publications: 11 monographs; 6 textbooks; 176 scientific articles in the fields of linear statistical models, estimation theory, regression analysis, and experimental design; 90 technical and popular articles on applications in geodesy, internal medicine, and metrology; 22 research reports.

Conference talks: 201

Editorial board member: *Mathematica Slovaca*; *Acta Univ. Palacki. Olomuc.*, *Fac. rer. nat., Mathematica*

Scientific society member: AMS, JČMF (Union of Czech Mathematicians and Physicists), Czech Statistical Society, Honorary Member of JSMF (Union of Slovak Mathematicians and Physicists)

Awards:

- Congress Premium (1st place in a world competition of young geodesists under 35 years of age) at the XI-th congress of *Fédération Internationale des Géomètres* in Rome, 1965;
- Plaque of the 25th anniversary of Geodesy and Cartography Department for merits in development of geodesy. Director of Slovak Institute of Geodesy and Cartography, 1979;
- Silver Aurel Stodola Honorary Plaque for merits in technology. SAS, 1981;
- Commemorative Medal of the 50th anniversary of Slovak Technical University for substantial merits in development of Slovak Technical University. Chancellor of Slovak Technical University, 1987;
- Medal for merits in development of mathematics and physics. Union of Slovak Mathematicians and Physicists, 1987;
- Corresponding member of Slovak Academy of Science, 1987;
- Commemorative Medal of the 200th birth anniversary of Jan Evangelista Purkyně, conferred to outstanding scientists. Slovak Academy of Science, 1988;
- Silver Medal for merits in development of mathematics and physics. Union of Slovak Mathematicians and Physicists, 1989;
- Honorary Member of Union of Slovak Mathematicians and Physicists, 1990;
- Golden Jura Hronec Honorary Plaque for merits in mathematical sciences. Slovak Academy of Science, 1991;
- Golden Medal of Faculty of Mathematics and Physics. Faculty of Mathematics and Physics, Komenský University, 1991;
- Bolzano Medal for merits in development of mathematical sciences. Czechoslovak Academy of Science, 1991;
- Silver Medal (pro merito) of Palacký University, 1996;
- *Nummum academiae memorialm tribuit (pro singularibus meritis de studiis scientiarum provehendis)*. Slovak Academy of Science, 2001;
- Golden Commemorative Medal of Faculty of Mathematics, Physics, and Informatics. Komenský University, 2001;
- Golden Medal (pro merito) of Palacký University, 2001;
- First Order Medal of Ministry of Education, Youth, and Physical Education, 2001;
- Honorary degree dr. h. c. at Slovak Technical University, 2002;
- Honorary membership in Scholarly Society of SAS, 2005;
- Celebrity of the year 2005. Slovak Academy of Science



This section contains collected photographs, and brief recollections and congratulations from Prof. Kubáček's co-workers, both contemporary and former, his colleagues and PhD students who had the privilege to meet him in person during his many active years in both scientific and educational worlds. In order to preserve the personal charm of texts, they appear here in their original language.

Thank you for your appreciation
Editors

Profesor RNDr. Ing. Kubáček, DrSc., dr. h. c. osemdesiatnik

Je neuveriteľné, že náš priateľ a kolega, Luboš Kubáček, sa už dožíva osemdesiatky. Dožíva sa jej v plnom zdraví, plný vitality a pracovnej energie.

Narodil sa 1. februára 1931 v Bratislave. Po absolvovaní štúdia geodézie na Slovenskej vysokej škole technickej v r. 1954 nastúpil ako vedúci výpočtového oddelenia v Geodetickom ústave v Bratislave. Zotrval tam osem rokov. Prostredie čísel a presných výpočtov ho silne motivovali a utvrdili v presvedčení, že bez matematiky – numerických metód a štatistiky – to ďalej nepôjde. Začal teda študovať popri zamestnaní a úspešne ukončil najprv v r. 1957 matematickú analýzu a v r. 1964 pravdepodobnosť a matematickú štatistiku, obe na Prírodovedeckej fakulte Univerzity Komenského v Bratislave. Neskôr prešiel do Slovenskej Akadémie Vied, do ústavu teórie merania. Tu sa Lubošovi podarilo, vďaka jeho obrovskému elánu, vybudovať silný kolektív štatistikov a matematikov, súčasť tzv. oddelenia teoretických metód. Z tohto kolektívu vyšli mnohí slovenskí štatistici a matematici, pôsobiaci na slovenských ale aj zahraničných akademických pracoviskách. Pravidelné stretnutia na seminároch ako magnet priťahovali mladých ľudí z celého Slovenska. Niektorí neváhali a pravidelne cestovali aj zo vzdialenejších miest.

V r. 1981 Luboš prešiel do Matematického ústavu SAV, kde v r. 1988–91 bol jeho riaditeľom, ale ani v období riadenia nepočul v úzkej spolupráci s kolegami a mladými študentami.

Počas celého pôsobenia v Bratislave zostal verný geodézii. Jeho teoretické práce, knižné publikácie a vedecké články, v ktorých sa venuje najmä riešeniu obtiažnych problémov v oblasti regresných modelov, a vďaka ktorým získal medzinárodné uznania, majú vždy priamu nadväznosť na konkrétne aplikácie. Napríklad veľmi výrazne prispel k rozvoju štatistiky teórie geodetických sietí. Neskôr sa však Lubošov aplikačný obzor ešte viac rozšíril – počas zhruba dvadsiatich rokov spolupracoval na riešení medicínskych a biomedicínskych problémov s 1. Internou klinikou v Bratislave. Spolu s manželkou Liduškou, ktorá neochvejne stála po jeho boku a bola mu životnou partnerkou a najbližšou spolupracovníčkou až do jej smrti, prispeli k riešeniu mnohých teoretických štatistických problémov v geofyzike. V tom období, v r. 1981 získal titul DrSc. a v r. 1991 bol menovaný profesorom.

Od roku 1994 Luboš pôsobí – pracuje, publikuje, prednáša a venuje sa naďalej veľmi aktívne výchove mladých matematikov – na Prírodovedeckej fakulte Univerzity Palackého v Olomouci. Aj tu sa jeho charizma naplno prejavila. Veľkou mierou sa zaslúžil o vybudovanie štatistickej skupiny v rámci aplikovanej matematiky.

Už počas pôsobenia v ÚTM SAV v Bratislave, Luboš pravidelne prednášal pravdepodobnosť a matematickú štatistiku na Komenského univerzite v Bratislave a venoval sa výchove aspirantov a mladých vedeckých pracovníkov. Jeho obetavosť a ochota pomáhať mladým nepozná hranice. Dodnes, hoci po X-tý krát dokáže tráviť mnohé hodiny s mladým adeptom alebo adeptkou a zasväcovať ich trpezlivo krok po kroku do základov pravdepodobnosti a štatistiky. Podarilo sa mu vyškoliť, či už na Slovensku alebo v Čechách najmenej 15 doktorandov, pričom v súčasnosti školí ďalších dvoch nádejných adeptov matematických vied.

Luboš dodnes nepoľavil v základnom teoretickom výskume – výsledky publikoval a stále publikuje nielen vo vedeckých časopisoch, ale v celom rade vedeckých kníh. Je autorom alebo spoluautorom 11 odborných kníh, z toho dva boli publikované renomovaným zahraničným nakladateľstvom. Je autorom alebo spoluautorom 6 skrípt a viac než 130 článkov v uznávaných medzinárodných vedeckých časopisoch. Odborné publikácie nájdeme ako v matematicky zameraných časopisoch tak aj časopisoch zameraných na geodéziu, chemometriu a lekársky výskum. Je autorom celého radu popularizačných článkov. Je členom niekoľkých redakčných rád vedeckých a odborných časopisov. Je nositeľom celého radu medailí, vyznamenaní a ocenení za celoživotnú prácu, za rozvoj matematickej štatistiky, aplikácií a popularizácie matematiky na Slovensku, v Čechách a vo svete.

Milý Luboš, do ďalších rokov Ti prajeme pevné zdravie, mnoho nových plodných myšlienok, nápadov a riešení, a najmä hodne spokojnosti v kruhu svojich kolegov, priateľov a najbližšej rodiny.

Marie Hušková a Júlia Volaufová



Olomouc 2003. Left to right: Lubomír Kubáček, Júlia Volaufová and Ludmila Kubáčková

Memories and congratulations

L. Kubáček sa narodil úmyselne pár mesiacov predo mnou čisto len preto, aby som sa nemusel hanbiť, že chodím na prednášky k mladšiemu kolegovi. Ale hneď na počiatku som zistil, že mu pri prednáškach ide aj o čosi iné ako o čistú matiku. Matika bola pre neho len nástrojom pre kreáciu komplexného post-modernistického umenia zloženého z vedy, maliarstva a hudby. Kým iní matici píšu na tabuľu vzorce rýchlosťou 12 km za hodinu, takže ich ľavou rukou musia hneď zotierať, lebo je tabuľa plná prv ako si poslucháč môže odhryznúť okraj maslového chleba s rybacím šalátom, Kubáček vzorce zásadne maľoval. Bolo vidieť, že keď začal šatírovať veľkú Δ , tak sme sa ho neraz pýtali, či to bude da Vinciho *Monna Lisa* alebo *Život* z Picassovej modrej periódy. Hlboko si vzdychol, pokýval hlavou a povedal: „Len Renoirova *Diana* začína veľkým *D*, to by ste mali vedieť.“ Potom sa akosi zase preinkaroval do sveta matiky a dopísal štvormetrový dôkaz rýchlosťou, o ktorej kozmonauti môžu aj dnes len snívať.

Na konci každého dôkazu vyšpúlil pery a z jeho úst sa vydral akýsi pískaný trilok, o ktorom som sa na počiatku domnieval, že ide o Mozartov *Turecký pochod*, ale o pár mesiacov som si bol istý, že ide o prvé dva takty klavírneho sóla zo Chopinovho koncertu e-mol, ktorý je trochu podobný, ale raz sa jeden študent nezdržal a povedal nahlas: „Capriccio!“ Kubáček súhlasne prikývol a riekol: „Správne, Liszt–Paganini, Capriccio a-mol. Z vás bude dobrý matematik!“

A tak nám vlieval do hlavy tzv. globálnu vedecko-umeleckú erudíciu, ktorú sa dnes rozumní politici pokúšajú presadiť na celom svete. Vraj turistika v Olo-mouci narastá a neraz tam vidieť anonymných ministrov kultúry z celého sveta, ktorí síce nenavštívia Kubáčkove prednášky – lebo by aj tak ničomu nerozumieli – ale dopytujú sa všetkých tamojších lekárov, či by bolo možné udržať Kubáčka na univerzite ešte aspoň 80 rokov. K tomuto želaniu sa kategoricky pridávam v mene všetkých lingvistov, ktorým nezištne pomohol, aj keď z nás nikdy nebudú ministri kultúry.

G. Altmann



Prof. RNDr. Ing. Luboš Kubáček, DrSc., dr. h. c. – spiritus mathematicus et statisticus

Prvý krát som sa stretol s menom profesora Kubáčka ešte ako študent v štvrtom ročníku. Vtedy sme ho oslovovali pán doktor a spolu s ďalšími jeho kolegami z vtedajšieho Ústavu teórie merania SAV nás učil ako externý učiteľ. Keď som 15. 8. 1972 nastúpil na Ústav teórie merania ako interný doktorand, tak tuším až tam som sa dozvedel, že pán doktor je pôvodne vyučený zememerač. Bolo to pre mňa prekvapenie o tom milšie, že aj môj otec bol zememerač a už ako malého chlapca ma brával na meračky. Na strednej škole som robil cez prázdniny figuranta na Geodézii v Šafárikove, malom mestečku na juhu stredoslovenského kraja, prakticky na hraniciach s Maďarskom. Takže som sníval, že aj ja sa stanem zememeračom. No nakoniec som sa stal matematikom a svojim spôsobom som bol medzi zememeračmi a prof. Kubáček bol mojim prvým šéfom. Vďaka nemu sa na Ústave teórie merania SAV vybudovalo vynikajúce pracovisko ma-

tematickej štatistiky a teórie pravdepodobnosti, ktoré dalo základ matematickej štatistiky na Slovensku a preslávilo ho po celom svete. Aj keď dnes mnohí vtedajší pracovníci pracujú po rôznych ústavoch SAV a na univerzitách, základ slovenskej štatistiky bol v jeho oddelení.

Raz som prišiel domov k rodičom do Šafárikova (dnes Tornaľa) a skočil som aj za otcom na Geodéziu. Jeho kolegovia sa ma hneď pýtali, kde som, čo robím. Tak som sa pochválil, že som ašpirant a robím v oddelení Dr. Kubáčka. „Kubáčka? Toho poznám, prednášal nám na kurzoch pre zememeračov, ale bola tam samá štatistika!“ Bol som šokovaný, ale zároveň aj hrdý, že aj v tom malom mestečku, ďaleko od Bratislavy poznali nášho doktora Kubáčka.



Bratislava 2005. Left to right: Lubomír Kubáček, František Štulajter, Gejza Wimmer, Andrej Pázman and Anatolij Dvurečenskij

Rokmi, keď sme potom sa stretli na Matematickom ústave, kde sme sedeli dokonca v jednej kancelárii, a kde sa Luboš stal riaditeľom, som sa tomu prestal čudovať, lebo jeho vklad do matematickej geodézie a matematickej štatistiky bol na slovenské pomery veľkolepý. Podarilo sa mu v dobrom rozvíriť aj vody na MÚ SAV, vyburcovať slovenskú matematickú štatistiku a aj matematiku takým spôsobom, že nielen Slovensko, ale aj celý svet uznanlivo prikyvuje hlavou. No okrem týchto odborných znalostí si nesmierne vážim jeho ľudský a otcovský prístup ku kolegom. On nikdy nehladal príčiny, prečo by niečo nemalo ísť, ale snažil sa nájsť spôsob, aby to išlo. Snažil sa povzbudzovať každého – počínajúc od najmladšieho. Takže nečudo, že spod jeho ruky vyšli mnohí matematici, ktorí sú dnes ozdobou matematického stavu nielen na Slovensku alebo Čechách.

Milý Luboš, keby mi niekto povedal, že keď sme oslavovali Tvoju päťdesiatku, tak dnes mnohí z nás budú už mať po šesťdesiatke, tak mu poviem to ešte

nebude dlho-dlho. Roky preleteli ako voda, a dnes ja už ako riaditeľ MÚ SAV, sa neraz pýtam sám seba pýtam, ako by to robil Kubáček, ako nás to učil.

Milý Luboš, dnes, keď vchádzaš do úctyhodného kmeťovského veku, by som Ti rád poďakoval za Tvoje rady a ľudský prístup, za rozvoj matematiky, výchovu doktorandov a poprial Ti všetko najlepšie, mnoho zdravia a šťastia. Nech Ti tvoje matematické pero nevyschne, píš aj obsiahle monografie, a teším sa na Tvoje návštevy na ústave a na Tvoje povzbudenia.

Živio, Luboš, ad multos annos! Tvoj

Tolo Dvurečenskij



Je to už neuveriteľných takmer 50 rokov, čo v lete 1961 Dr. Bolf poslal Sylvii Martiny (Pulmannovú) a mňa na Ústav geodézie a kartografie za istým Ing. Kubáčkom požičať si presný planimeter. Milý pán, ktorého sme tam stretli, sa stal zakrátko našim kolegom na SAV a po roku 1969 i našim vedúcim oddelenia. Ale ešte predtým, v jednej miestnosti pilne pracovali a konzultovali štyria budúci riaditelia: Kubáček, Kukuča, Štuler a Marčák. Pracovitost' vtedajšieho Ing. Kubáčka bola neuveriteľná: okrem každodennej práce na ústave ešte štúdium na druhej VŠ (pravdepodobnosť a štatistika), aspirantúra v skrátrenom termíne, výskum, ktorý viedol ku 1. miestu v európskej súťaži mladých geodetov, výchova dvoch detí s manželkou, ktorá sa snažila v ničom vedecky nezaostať. Dodnes ho vidím, keď som mu naškrtol, že v istej situácii môže byť istá chyba merania negausovská, okamžite spísal nemecký článok do *Vermessungstechnik* (1964), z ktorého zažltnutých strán dnes vykuká aj moje, nie plne zaslúžené spoluautorstvo. A myslím si, že dnešný Dr. Karovič a iní majú podobné spomienky na jeho vtedajšiu ústretovosť.

Avšak skutočné obdobie našich kontaktov a spolupráce začalo po r. 1969 po mojom návrate z Dubny. Jednak spoločná práca na našej (ale hlavne jeho) knihe *Štatistické metódy v meraní*, v ktorej si dodnes so záľubou listujem, jednak určité, čiastočne spoločné pionierske kroky pri počiatocnom budovaní štatistickej skupiny na Ústave merania SAV. Striedavo sme driemali na nekonečných seminároch prof. Winkelbauera z Prahy (Clifordtove algebry!!), počúvali sme prednášky docentov Josifka, Macheka a iných z Prahy, profesorov Vinczeho a Revesza z Budapešti, ktorých návštevy na SAV inicioval, spoločne v trojici s dnešným doc. Štulajterom sme zo začiatku chodili ako traja králi po pražských a budapeštianskych vedeckých pracoviskách. Alebo lúskali sme Partassarathyho knihu *Miery na metrických priestoroch*, či diskutovali o využití metód optimalizácie experimentov. Je to až neuveriteľné, že v politicky značne napätej dobe, keď (slovami Lasicu a Satinského) „za dverami zúrila metelica“ vo vnútri oddelenia panovala priateľská a pritom pracovná atmosféra. Buď Ti Luboš za to vďaka.

Osemdesiate roky už boli turbulentnejšie. Po takmer súčasnom obhájení veľkých doktorátov, „boli sme odídení“ na iný ústav SAV, a Luboš Kubáček mi unikal do vyšších sfér členov korešpondentov a riaditeľov. A tiež do neprehľadných vzorcov Raovskej maticovej štatistiky. Keď začiatkom 90. rokov ja som odišiel na univerzitu a o 3 roky prof. Kubáček do Olomouca, kontakty ešte zoslabli. A myslím si, že nie mojou vinou. Ale život je už taký.

Ja môžem dnes len obdivovať, že už tretíkrát, po skupine na ÚM SAV a po skupine na geodézii na STU, sa Ti Luboš podarilo opäť stmeliť a motivovať v Olomouci kolektív mladých ľudí, ktorí majú radi vedu. A splnil si do písma všetky požiadavky Hemingwaya, ktoré sa Ti tak páčia a ktoré hovoria, že k úspechu treba mať talent, byť pracovitý a dožiť sa, a želám Ti, teraz hlavne v tom poslednom, ešte dlhé napĺňanie.

Andrej Pázman



Lubomír Kubáček with Andrej Pázman and Ivica Mišíková



Milý Luboš,

Dovoľ mi, aby som Ti pri príležitosti Tvojho veľkého sviatku čo najúprimnejšie zo srdca pogrataloval a poprial veľa, veľa pevného zdravia, pohody, šťastia a spokojnosti. Pri tejto krásnej príležitosti Ti chcem poďakovať za všetko, čo som od Teba dostal. Podstatne si ovplyvnil moju profesionálnu životnú dráhu a jej smerovanie už počas môjho štúdia a aspirantúry pod Tvojim vedením. Nesmiernym žriedlom vedomostí, skúseností a prístupov k riešeniu problémov boli dni a hodiny, ktoré som mohol stráviť pri Tebe v kancelárii, pri konzultáciách a pri rozhovoroch. Z Tvojich postojov a z Tvojho správania sa som pochopil, čo je skutočné pracovné nasadenie, pracovitosť, húževnatosť. Ale pocítil som aj to, čo znamená mať rád matematiku a štatistiku. Snažím sa moju vďaku za toto všetko splácať svojim študentom, poslucháčom.

Živjo, živjo, živjo!

Gejza Wimmer



Probastat 1991. Lubomír Kubáček, Júlia Volaufová and Gejza Wimmer (right) with colleagues



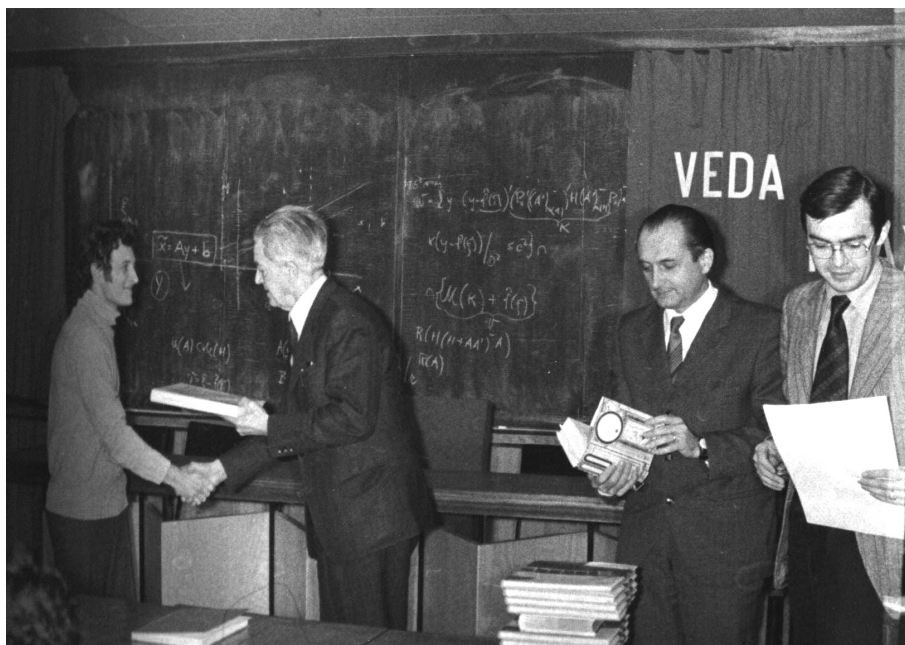
Na Ústav teórie merania SAV som ako absolvent techniky prišiel v roku 1970. V priebehu prvého roku môjho študijného pobytu som zistil, že napriek môjmu výbornému prospachu na škole mi vedomosti z matematiky na vážnejšiu vedeckú prácu asi stačiť nebudú. Dr. Kubáček bol v roku 1972 jeden z dvoch mojich nadriadených, ktorý mi poradil ako takto pocitovaný deficit nahradiť, a s jeho „požehnaním“ som teda súčasne s aspirantúrou začal aj externé štúdium numerických metód na MFF UK.

Nasledujúcich päť rokov bolo naozaj mimoriadne namáhavých a viackrát som bol na pokraji rozhodnutia vzdať to. Ľuboš, ako vedúci oddelenia, v ktorom som žil, mal však chválaboju obrovský zmysel najmenej pre dve veci: (1) ponechanie veľkej slobody pri mojom permanentnom prepínaní medzi aspirantskými a školskými povinnosťami, a (2) ochotu poradiť mi v akomkoľvek matematickom, či ľudskom probléme. Po úspešnom skončení aspirantúry i ďalšieho univerzitného vzdelania v roku 1977, som v priebehu ďalších tridsiatich rokov dlhodobo zakotvil na troch zahraničných výskumných pracoviskách a všade sa ukázalo, že päť rokov „galejí“ na začiatku malo svoj hlboký zmysel, a nikdy som tento čas neolutoval.

Milý Ľuboš, pri príležitosti Tvojho významného životného jubilea by som Ti chcel, po rokoch, znovu poďakovať za Tvoje nesmierne cenné „navigačné“ úsilie, ktoré do značnej miery zmenilo moje elévske predstavy o výskume a určilo moju pracovnú životnú púť.

Drž sa ešte dlho v zdraví!

Ivan Bajla



A prize-awarding ceremonial in “Science to Practice” competition, Bratislava 1979.
Left to right: Ivan Bajla, Ludevít Kneppo, Lubomír Kubáček and Pavol Tekel



Skromnosť zvaná vzdelanosť

Vzdelanosť Luboša Kubáčka. Meraná nielen počtom a kvalitou diplomov (dve vysoké školy, či veľký doktorát), ale najmä jeho postojom k vede a k životu.

Na Univerzite Komenského sme začali pôsobiť zhruba naraz, koncom 60. rokov. Po revolučnom roku 1968 a následnej ruskej invázii sa katedra teórie pravdepodobnosti a matematickej štatistiky prakticky rozpadla. Pod vedením Tibora Neubrunna sme ju začali budovať prakticky znovu a Luboš bol jedným z jej základných kameňov. Výsledkom sú desiatky úspešných absolventov tohto odboru uplatňujúcich sa tak vo výskume medzinárodného dosahu, ako aj v širokom spektre aplikácií.

Druhým miestom vedeckej tvorby v uvedenom smere bolo Lubošovo oddelenie na Ústave teórie merania SAV. Pre mňa bolo potešením zúčastňovať sa aspirantských skúšok na tomto ústave. Tu vznikla aj tradícia Probastatov. Jednou z veľkých Lubošových zásluh v druhej polovici 80. rokov bola jeho podpora začínajúceho výskumu v oblasti fuzzy množín: na Probastate 1987 mi prideliť cyklus prednášok o fuzzy množinách.

Bolo prirodzené, že po revolúcii 1989 Univerzita Komenského navrhla Lubošovi profesúru. Pravdaže, Luboš nebol ani docentom. Bolo dojímavé sledovať, ako najlepší československí odborníci na oťocku napísali posudky k docentúre i profesúre. Posudky svedčiacie nielen o Lubošových významných výsledkoch, ale aj o jeho vzácných ľudských kvalitách.

Beloslav Riečan

Vážený pán profesor, milý Luboš.

Moju životnú cestu významne ovplyvnili 4 osobnosti. Jednou z nich si Ty. V roku 1963, keď si prišiel pracovať do Ústavu teórie merania SAV, zaoberal som sa so spracovaním a vyhodnotením súborov nameraných hodnôt. Na pokyn dr. J. Bolfa sme spolu prediskutovali problém. Voviedol si ma do tajov matematicko-štatistického spracovania experimentálnych údajov. Odvtedy sme sa zblížili, nielen pracovne, ale aj ľudsky. Chodil som na Tvoje semináre, diskutoval problémy. Vážim si u Teba to, že pri návrhu experimentu a spracovaní jeho výsledkov sa na problém získania nameraných hodnôt pozeráš ako geodet, čo bola Tvoja pôvodná profesia, a spracovanie experimentálnych dát manažuješ ako naslovovaný matematik. Za získaným číslom cítiš vynaloženú námahu experimentátora, čo u matematikov nebýva bežné.

Nedá mi, aby som z hľadiska mojich predošlých funkcií v ústave a v SAV nevyzdvihol Tvoj úžasný podiel pri formovaní matematickej skupiny v Ústave teórie merania SAV. Táto skupina sa stala vďaka Tebe i Tvojej manželky Ľudmily, ktorá nás žiaľ privčias opustila, ikonou ústavu, presahujúcu hranice Slovenska. Tvoj vplyv cítim v tejto skupine ešte stále, hoci mnoho rokov pracuješ inde.

Si mojím vzorom nielen ako vedec, ale najmä ako človek. Tvoj charakter, vzťah k ľuďom, sú pre mňa štandardom, ako má vyzeráť vysokoškolský profesor. A ak sa mám k niekomu vyjadriť, či si zaslúži prejsť inauguračným konaním, vždy ho porovnám s Tebou.

Luboš, som nesmierne šťastný, že mi osud doprial s Tebou sa v živote stretnúť a mnoho sa od Teba naučiť. Želám Ti dobré zdravie, šťastie a radosť nielen v kruhu Tvojej rodiny, ale aj medzi Tvojimi doktorandmi.

Úprimne Tvoj

Karol Karovič



Probstat, Smolenice 1994. Lubomír Kubáček with his wife Ludmila, Beloslav Riečan and Karol Karovič

Milý Luboš,

spomienka na spoločný seminár z teórie miery patrí k najlepším spomienkam v mojom profesionálnom živote. Veľmi povzbudzujúce bolo Tvoje tvrdenie, že si stačí veci zopakovať 16 krát, aby si ich človek natrvalo zapamätal (i keď v neskorších rokoch si 16 zmenil na 160). Tvoje tvrdenie „Vedecký pracovník musí byť schopný prežiť akúkoľvek formu blamáže“ mi pomáhalo a stále pomáha prežiť. K Tvojmu jubileu Ti srdečne blahoželám, a prajem Ti ešte veľa šťastných a tvorivých rokov.

Sylvia Pulmannová



Institut of Measurement Science SAS, Bratislava 1992. Front line (l-r): Lubomír Kubáček, Miloslav Duchoň, Andrej Pázman. Back line (l-r): Júlia Volaufová, Gejza Wimmer, František Štulajter, Sylvia Pulmannová



Milý Luboš,

dovoľ aby som Ti čo najsrdečnejšie zaželel všetko najlepšie k Tvojmu významnému životnému jubileu a pri tejto príležitosti poďakoval za Tvoju neustálu podporu a pomoc. Ďakujem, že si sa stal zdrojom motivácie a inšpirácie v mojej vedeckej práci – jednoducho vzorom nadšeného vedca, skromného a dobroprajného človeka a napokon aj veľmi dobrého priateľa. Patríš do úzkeho kruhu mojich najvýznamnejších učiteľov, ktorí prispeli k rozhodnutiu zotrvať s matematickou štatistikou, ako zaujímavou a fascinujúcou vedeckou disciplínou, na celý život. Bol si to práve Ty, kto ma po skončení vysokoškolského štúdia nasmeroval do Ústavu merania Slovenskej akadémie vied a jeho Oddelenia teoretických metód, ktorého zakladateľom a dlhoročným vedúcim si bol práve Ty. Aj vďaka Tvojej vytrvalosti a osobnej rozvahe a odvahe sa podarilo prekonať aj zložitejšie obdobia formovania vedeckej základne matematickej štatistiky na Slovensku. Oddelenie dodnes nesie Tvoju pečať a hrdí sa tým, že patrí medzi významné

centrá slovenskej matematickej štatistiky, kde pôsobili a boli vychovávaní významní slovenskí matematickí štatistickí už počas štyroch generácií.

Milý Luboš, želim Ti predovšetkým pevné zdravie, veľa pohody a spokojnosti v kruhu rodiny, priateľov a kolegov. Verím, že jasná iskra v Tvojich očiach nám bude ešte veľmi dlho prinášať radosť a pomáhať nachádzať nové perspektívne smery vedeckého bádania v oblasti matematickej štatistiky.

Viktor Witkovský



Olomouc 2003. Left to right: Viktor Witkovský, Lubomír Kubáček, Júlia Volaufová, Gejza Wimmer



Vážený pán prof. Kubáček, milý Luboš!

Pri príležitosti Tvojho životného jubilea by som Ťa veľmi rád pozdravil a zaželel Ti ešte veľa zdravia a elánu do ďalšieho života. Bolo pre mňa veľkou ctou byť Tvojim kolegom v práci počas celých šesnástich rokov. Cením si to o to viac, že si stál na začiatku mojej pracovnej kariéry a bol si mojim vzorom. Tvoja pracovná zanietenosť a láska k matematike sa prenášala aj na Tvojich kolegov, medzi ktorých som patril aj ja. V Tvojom prípade sa úplne potvrdilo, že „zapáľovať môže len ten, kto sám horí“. Viem, že táto schopnosť Ti zostala až doteraz a verím, že bude trvať ešte dlho, čo Ti z celého srdca želim. K tomu ešte pevné zdravie, spokojnosť a veľa radosti a potešenia z Tvojich synov a vnúčenciev.

Fero Štulajter



Zdravím Ťa Luboš!

Ako Tvoj niekdajší podriadený Ti k Tvojmu peknému životnému jubileu želim všetko dobré a nech Ti slúži zdravie.

Fero Rublík

Milý Luboš!

Blíží sa Tvoje okrúhle životné jubileum, a tak veľmi rada Ti blahoželám k tomu, čo si dokázal, k tomu, čo sa Ti v živote podarilo. Prajem Ti pevné zdravie, čulú myseľ, veľa šťastia, veľa radosti a aj veľa ďalších skvelých úspešných nápadov a výsledkov v práci.

Neviem či si uvedomuješ, ale je tomu už 38 rokov, čo som nesmelo zaklopala na Tvoje dvere na Ústave teórie merania SAV s nádejou, že budeš mojim vedúcim diplomovej práce. Stalo sa, a veru odvtedy si jedným z najvýznamnejších ľudí v mojom živote. S potešením sa hlásim ku „Kubáčkovej škole“, ktorá má najmä v oblasti štatistiky lineárnych modelov už mnohých odchovancov nielen na Slovensku a v Čechách, ale aj vo svete.

S obdivom spomínam na naše pravidelné stredajšie, tzv. riešiteľské semináre (na ktoré sme sa poniektorí pripravovali celé dni a dlho do noci, len aby sme si hanbu nespravili), a na ktorých si dokázal s úžasnou výdržou a elánom prednášať niekedy aj štyri hodiny bez oddychu. Bolo mnoho dní, keď sme sa v Tvojej kancelárii do úmoru, s obrovskou vytrvalosťou z Tvojej strany, snažili spolu dokázať nejaké maticové tvrdenie, ktoré sa nechcelo poddať. Naučil si ma toho nesmierne veľa. Si mi vzorom svojou neuveriteľnou húževnatosťou, vytrvalosťou a schopnosťou byť tak zaniatený pre konkrétny problém, že perfektne nakazíš celé okolie. Pritom to robíš s nenapodobiteľným taktom a človečnosťou. Do ďalších úspešných rokov prajem naďalej taký krásny ‘takt’ nielen Tvojmu srdcu, ale celej Tvojej bytosti.

Milý Luboš, s máličko zarosenými očami Ti srdečne blahoželám!

Julka Volaufová



Probstat, Smolenice 1994. Left to right: Lubomír Kubáček, Júlia Volaufová, Beloslav Riečan, Peter Mederly, Calyampudi Radhakrishna Rao, Baltazár Frankovič



European Meeting of Statisticians, Berlin 1988. Lubomír Kubáček with Jurgen Kleffe and Júlia Volaufová



Jednoho dne mi prof. Bukovský řekl, že mi vyjednal kandidaturu v Bratislavě a že mým školitelem bude prof. Kubáček. Samozřejmě, pokud vše schválí Strana, však víte která (a pokud nevíte, blaze vám). Kromě toho, že jsem byl silně skeptický ohledně toho schválení (vždyť ve Městském výboru KSS seděli pořád titíž, kteří mého otce vyhazovali z univerzity), vůbec jsem netušil ani u jména potenciálního školitele, o koho jde. Zázrak však nastal, byl jsem schválen, a tak jsem jel do Bratislavy na SAV-ku seznámit se. Seznamování probíhalo rychle, Luboš nikdy neměl tendenci mrhat časem. Několik hodin na mě chrlil své představy a záměry, problémy, které řeší a které chce řešit, co se děje ve výzkumu ve světě, matematické zajímavosti s tím spojené, jak to navazuje na různé aplikace, co má a co nemá cenu z hlediska aplikací, co všechno při výzkumu zažil, jaká je jeho filozofie matematiky atd. atd. Objem mé hlavy rychle rostl. Jedno však bylo od začátku jasné – je přede mnou mimořádná osobnost, která netrpí žádnou nadutostí, plně žije matematikou, a od které se mohu hodně naučit. Tento dojem se při dalších setkáních jen utvrzoval. Bývaly to náročné dny, když jsem ráno vypadl z vlaku po dost mizerném a krátkém spánku v lehátkovém voze a Luboš do mě s plnou vervou bušil celý den až do večera, aby mě něco naučil. Byl tak plný matematiky, že jsem měl pocit, že by s ním měla chodit sekretářka, která by zapisovala všechno, co říká, a pak by se to vydalo jako učebnice. Já bych si to potom postupně četl a snažil se to pochopit. Takhle naráz jsem se v tom dost ztrácel. Zlomilo se to až když jsem měl možnost být s ním v Bratislavě celý měsíc. Tehdy vznikl základ mé disertační práce. Dnes mě pouze mrzí, že jsem ho neuvedl jako spoluautora do prvního článku, na němž měl nezanedbatelný podíl. Prostě mě to nenapadlo. Bylo by to bývalo spravedlivé a mohl bych se dnes

chlubit, že s ním mám společný článek. Luboš byl opravdu skvělým školitelem a oblíbil jsem si ho i jako člověka. Naše názory často souzněly. Jsem šťasten, že jsme nezůstali jen u profesionálních vztahů, ale mohli se časem stát i přáteli. Léty pak toto přátelství neupadá, ale upevňuje se.

Tak ještě mnoga ljeta, Luboši!

Ivan Žežula



Moje první setkání s panem profesorem Kubáčkem

V roce 1995 jsem organizovala na Čeříнку u Jihlavy letní školu Matlab'95. Na tuto letní školu jsme pozvali pana profesora Kubáčka a jeho manželku Lidušku. Pana profesora předcházela pověst vynikajícího statistika a já jsem měla obavy, zda bude pan profesor spokojen s programem, organizací a poněkud skromným ubytováním. Ale mé obavy byly zbytečné, neboť se ukázalo, že profesor Kubáček je nesmírně příjemný a laskavý člověk a je vynikajícím společníkem. Spolu s manželkou podstatně přispěli k v tvůrčí atmosféře, která se na této letní škole vytvořila. V dalších letech jsme spolupracovali v rámci grantů, při přípravě a realizaci přednášek pro odbornou veřejnost. Profesor Kubáček se vždy zúčastnil letních škol Datastat a konferencí, které jsme pořádali. Velmi si také cením toho, že kdykoliv jsem ho požádala o členství v komisi pro státní doktorské zkoušky nebo při oponentním řízení, vždy mně vyhověl. Jsem mu také zavázána za jeho podporu při mém profesorském řízení. Ještě bych chtěla připomenout, že díky duchapřítomnosti pana profesora nedošlo k tragické nehodě během zpáteční cesty z letní školy biometriky. Pan profesor Kubáček je pro mě nejen vynikajícím odborníkem, ale také vzácným a laskavým člověkem.

Milý pane profesore, k Vaším narozeninám Vám přeji především hodně zdraví, pohody a stále tolik neutuchající energie a nadšení pro vědeckou práci zejména s mladými pracovníky a studenty.

Ivana Horová



S panem profesorem Kubáčkem jsem se poprvé setkala koncem roku 1981, kdy jsem jako začínající asistentka na Katedře aplikované matematiky Přírodovědecké fakulty brněnské univerzity skládala doktorskou zkoušku z pravděpodobnosti a statistiky na Matematicko-fyzikální fakultě Univerzity Komenského v Bratislavě. Pan profesor byl členem zkušební komise a pamatuji si, že mi položil otázku týkající se diskriminační analýzy. Mnohorozměrné metody se mi líbily, zkouška proběhla hladce. Velmi na mě zapůsobilo, jaký obrovský nadhled nad problematikou, která se mi tehdy jevila docela obtížná, pan profesor Kubáček projevil. Tento nadhled jsem pak obdivovala při každém jeho vystoupení na různých akcích, ať už to byly letní školy ROBUST, MATLAB a DATASTAT, konference ODAM či statistické kurzy pro odbornou veřejnost, které jsme pořádali v polovině 90. let. Myslím, že pan profesor svým pojetím přednášek dokáže přesvědčit posluchače o tom, že matematická statistika je báječná věda a rozhodně stojí za to se jí věnovat.

Díky, pane profesore!

Marie Budíková

Je mi velkou ctí a potěšením v rámci konference ODAM u příležitosti životního jubilea pana prof. RNDr. Ing. Lubomíra Kubáčka, DrSc., Dr.h.c. vyjádřit svoji úctu a vděčnost k jeho významné vědecké a pedagogické činnosti.

V době svého působení na katedře aplikované matematiky jsem se zabýval matematickou statistikou a účastnil jsem se všech přednášek profesora Kubáčka při jeho návštěvách v Brně a konferencích DATASTAT. Jeho přednášky měly vynikající vědeckou a metodickou úroveň, obsahovaly mnoho nevyřešených úloh z praxe a načrtávaly další perspektivní oblasti ve vývoji v oboru matematické statistiky.

Velmi oceňuji jeho příznivý postoj k mé snaze zabývat se matematickou statistikou a mému algebraickému přístupu v tomto oboru. Profesor Kubáček měl pro tento přístup porozumění, povzbuzoval mě a poskytoval mně užitečné informace o různých problémech z matematické statistiky a doporučoval vhodnou literaturu. Tohoto jeho postoje a podpory mého úsilí si vysoce cením a jsem mu za ně vděčný.

Ladislav Skula



ODAM, Olomouc 2004. Front line left to right: Pavla Kunderová, Zuzana Prášková, Marie Budíková, Gejza Wimmer, Lubomír Kubáček, Ladislav Skula



Vážený pane profesore, milý Luboši.

Několik desítek let se Kubáčkovcům motám do života. Každé prožité chvílky s Liduškou i s Tebou si upřímně vážím. Nechtěl bych opakovat jenom náš jediný společný životní nebelec. Oběma nám šlo o život a tys nás zachránil.

Přeji Ti můj školiteli hodně a hodně zdraví, trošku štěstíčka a splnění všech Tvých snů a plánů.

Ivoš Moll



Summer School of Biometrics, Lednice 2010. Left to right: Lubomír Kubáček, Ivo Moll and Zdeněk Pospíšil



*„Hlava se mi z toho točí,
prosím tedy o radu.“*

S panem profesorem Kubáčkem Lubomírem jsem se poprvé setkal v létě 1997. V tomto období jsem nastoupil v Olomouci na funkci vedoucího metrologie *Laboratoře metrologie ionizujícího záření*. Jedním z mnoha úkolů tohoto pracoviště je stanovovat nejistoty výsledků měření při ověřování měřidel, jejich navazování, při expertizách, apod. V tomto oboru záření jde o nelehký úkol. Viděl jsem, že je naprosto nezbytné seznámit se alespoň se základy matematické statistiky. Po domluvě s p. prof. Kubáčkem za ním chodím na ústní konzultace.

Nevím, zda jsem vůbec kompetentní pana profesora hodnotit. Přesto bych rád, jestli mohu, posoudil, že se jedná o výjimečně nadaného a odborně zdatného člověka, což vyžaduje z mé strany hlubokou úctu a ocenění. Při konzultacích se neustále ze strany pana profesora setkávám zejména s velkou vstřícností, solidností, spolehlivostí, laskavostí a lidskostí. Imponuje mi jeho široká vzdělanost, obecný rozhled a pedagogický takt. S takovými pozitivními lidskými vlastnostmi jsem se u jednoho člověka ještě nesetkal. Vždy, když po konzultaci od pana profesora odcházím, mám pocit lidského pohlazení.

Proto Vám, pane profesore, z celého srdce děkuji za Vaš hluboce lidský postoj, trpělivost a celkový pozitivní přístup k životu. Díky tomuto Vašemu přístupu se mi statistika stala opravdovým koníčkem. Přeji Vám mnoho osobní spokojenosti a hlavně pevné zdraví. Ještě jednou mnohokrát děkuji.

Václav Hora

Vážený a milý pane profesore, děkujeme – zůstaňte!

Jan Andres



Professors at the Department of math. analysis and appl. math. with colleague, Olomouc 2003. Left to right: Lubomír Kubáček, Svatoslav Staněk, Irena Rachůnková, Milan Tvrďý, Jan Andres



Jak jsem nevěřila panu profesoru Kubáčkovi

Dlouhá léta jsem byla na katedrách matematiky přírodovědecké fakulty UP v Olomouci jediným statistikem. Výhodou bylo, že jsem byla nejlepší, nevýhodou nutnost přednášet mnoho kurzů a zkoušet velké počty studentů. Velmi těžká ale byla osamocenost ve vědecké práci. Nepatřila jsem do žádného kolektivu vědeckých pracovníků, neměla jsem nikoho, kdo by mne inspiroval, pomáhal mi třídit myšlenky, radil mi s matematickými problémy.

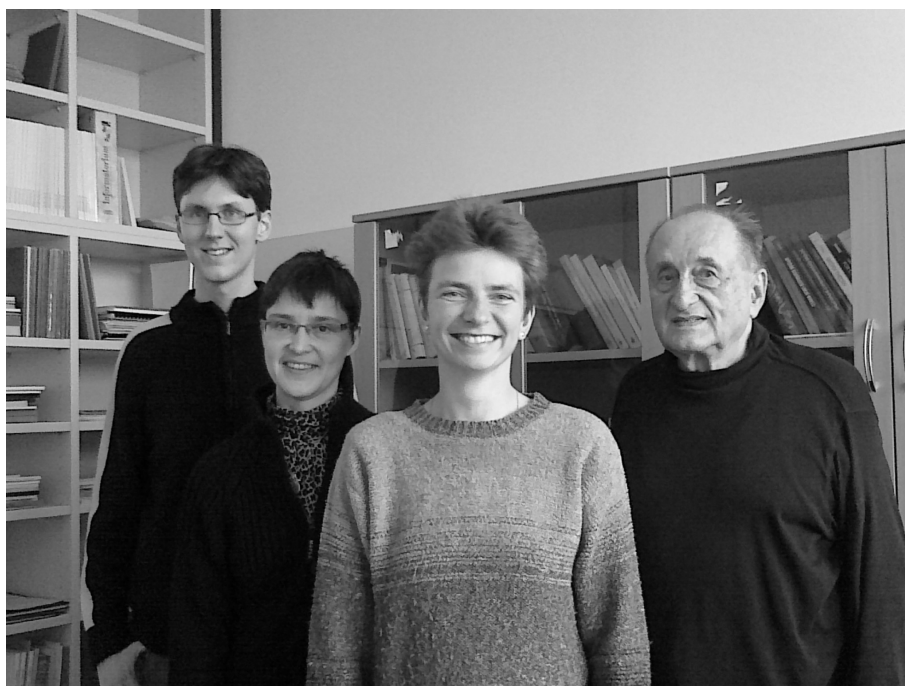
Vše se změnilo po příchodu pana profesora Kubáčka a jeho vzácné paní na olomouckou univerzitu. Pan profesor se mi obětavě věnoval, seznamoval mne s problematikou lineárních statistických modelů, radil mi, co studovat, o čem přemýšlet. Velmi ochotně mi poskytoval konzultace k problémům a úskalím, na které v matematice člověk vždycky narazí. Pan profesor tvrdil, že se do deseti let budu habilitovat, že postupné drobné úspěchy a pravidelná práce přinesou po letech ovoce. Nevěřila jsem mu! Těšilo mne, když jsem napsala článek do vědeckého časopisu, ale docentura? Kdo by tomu věřil v mém věku!

Jaký je konec mého vyprávění? Pan profesor měl ve všem pravdu! Patřila jsem do mladého kolektivu statistiků, který se kolem pana profesora Kubáčka vytvořil, a stala jsem se docentkou. Pane profesore díky a ještě jednou díky!

Pavla Kunderová

Říká se, že mít dobrého školitele je pro doktoranda velká výhra. V případě profesora Kubáčka to platí stonásobně. Každé z jeho *dítek* (tento výraz snad dostatečně zdůrazňuje fakt, že pan profesor je svým doktorandům více otcem než jen pouhým tutorem) potvrdí, že začátky nebývají lehké. Desítky hodin konzultací, po kterých aspiranta bolí hlava a má pocit, že *tohle* prostě nikdy nepochopí a zůstane totálním hlupákem, se pomalu mění v příjemná setkávání a nadšené rozkrývání nových obzorů. Pan profesor ohromí nejen svými odbornými znalostmi, ale i širokým životním rozhledem a zkušenostmi, o které se neváhá podělit. Za to všechno mu náleží od jím vychovaných dítek velký dík.

Eva Fišerová a Jana Vrbková



Olomouc 2011. Left to right: Karel Hron, Jana Vrbková, Eva Fišerová, Lubomir Kubáček



Výrok, který se přisuzuje jednomu známému rakouskému matematikovi, že *matematika je jako past na myši (chytí a již nepustí)*, jsem si připomněl při prvním osobním setkání s panem profesorem L. Kubáčkem a s jeho paní. „Kubáčkovi jsou v té pastí“, říkal jsem si. Ale pak jsem si hned uvědomil, že „oni ji musí mít velice komfortní a důmyslnou, když se s takovou chutí v ní, i na chvilku mimo ni, pohybují“. Později jsem přišel i na to, že s tou pastí se Kubáčkovi dostali do čtvrtého rozměru, kam mohou za tvrdou službu jen bohem vyvolení.

Bohu teď děkuji za to, že měl tak dobrou volbu.

Zdeněk Půlpán



Prof. Kubáček je pevný bod v mém profesním vesmíru, autorita, charizmatická osobnost, moudrý a čestný muž, dokonalý džentlmen a navíc člověk, kterého mám ráda. Velmi si cením našich setkání a rozhovorů – jsou pro mne velkou oporou, potvrzením správnosti cesty, obohacují mé myšlenky, dodávají mi energii a přinášejí jas a radost do života.

Pane profesore, k Vaším narozeninám Vám snad nejlépe popřeji slovy, která používáte při podobných příležitostech Vy – přeji Vám všechno dobré!

A přeji Vám to z celého srdce.

Jana Talašová



Olomouc 2008. Left to right: Svatoslav Staněk, Pavla Kunderová, Karel Hron, Lubomir Kubáček, Irena Rachůnková, Radomír Halaš, Jana Talašová



Vážený pán profesor, je mi veľkou ctou a potešením, že sa môžem aspoň takto na diaľku pripojiť k množstvu gratulantov k Vášmu životnému jubileu. Ste a verím, že aj budete, inšpiráciou pre Vašich študentov, kolegov a priateľov doma, aj vo svete. Som rada, že som mala to šťastie spoznať Vás. Učím sa od Vas umeniu, ako viesť mojich študentov k zodpovednej, zmysluplnej a radosťou naplnenej tvorivej práci, či už doma, na Slovensku, alebo v mojom terajšom pôsobisku v Texase. Vďaka ľuďom, ako ste Vy, je veda dokonalejšia a život krajší. Želám Vám predovšetkým veľa dobrého zdravia a pripájam “a big hug of Texas size”. Vivat academia, vivat professores! Vivat Profesor Kubáček!

Slávka Bodjanová

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