Olomoucian Days of Applied Mathematics

ODAM 2019

BOOK OF ABSTRACTS

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Department of Mathematical Analysis and Applications of Mathematics
Faculty of Science, Palacký University Olomouc

Preface

Dear Friends and Colleagues,

Welcome to the international conference "Olomoucian Days of Applied Mathematics 2019" (ODAM). This forum aims at bringing together researchers in mathematical modelling as applied to various scientific fields to discuss current challenges and opportunities brought forward by practical problems. The conference is intended as a forum for presenting new developments and applications of probability theory and statistics, numerical mathematics and optimization, and other branches of applied mathematics.

The conference ODAM came into existence through the initiative of Professor Lubomír Kubáček in 1999, an outstanding figure in Czech and Slovak mathematical statistics. He took up the tradition of seminars in applied mathematics held at the Department of Mathematical Analysis and Applications of Mathematics at Faculty of Science of Palacký University in Olomouc and established a tradition of friendly meetings of applied mathematicians, alternately focused on mathematical statistics and fuzzy sets on the one hand, and mathematical modelling on the other. Since 2011, the conference ODAM has been organized biennially as a full-scale international conference. This year the scientific program contains also six invited lectures of renowned personalities in the field. The social program includes a dinner in a local brewery, a dinner in the green heart of Olomouc, and a commented visit of the Botanic Garden, just few steps from the main entrance of the Faculty of Science which serves as the conference venue.

We wish you a productive, stimulating conference and a memorable stay in Olomouc.

Eva Fišerová and Karel Hron, editors

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Olomouc 2019

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Abstracts

A Robust Adaptive Modified Maximum Likelihood Estimator for the Linear Regression Model

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The least squares (LS) estimators are widely used in regression analysis because of their easy computation. They have good statistical properties when the distribution of the error terms is normal, i.e. they are the best linear unbiased estimators. However, they lose their efficiencies in presence of outliers. Therefore, robust statistical methods, which are insensitive to the outlying observations, are employed as alternative to LS method in case of outliers. Among the robust estimators for the linear regression model, we here consider the adaptive modified maximum likelihood (AMML) estimators proposed by Donmez [1]. As their name refer, the AMML estimators are essentially adaptive versions of the modified maximum likelihood (MML) estimators, see for example [2,3]. The AMML estimators are also easy to compute since they are defined by explicit formulas. Although the AMML estimators are insensitive to y outliers, they are not robust to x outliers. In this study, we suggest the robust AMML (RAMML) estimators which are obtained by giving extra weights to the predictors. We conduct a simulation study to compare the performances of the RAMML estimators with some existing robust estimators such as MM, LMS, LTS and S by using different simulation schemes. The results demonstrate that the RAMML estimators outperform the others according to the mean squared error (MSE) criterion in most of the cases.

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Behaviour of Higher Order Approximations in the Cox Proportional Hazards Model

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Survival analysis is applied in a wide range of sectors, such as medicine, economy, technology and others. Its main idea is the evaluating of the time until the occurrence of an event of interest and the description of the effect of some particular covariates on survival time is based on the Cox proportional hazards model. Moreover, the statistical significance of the effect of considered covariates is verified by the likelihood ratio test, the Wald test, or the score test. The mentioned tests represent the first-order approximations which are asymptotically equivalent; lead to the numerically different results in applications according to available data. In addition to that, higher-order asymptotics based on Barndorff-Nielsen and Lugannani-Rice approximations is used for more accurate results. Comparison of the size, power, and adjusted power of these tests for samples with small size is performed on simulated datasets in dependence on the distributions of baseline hazard functions, various proportion of right censored data and the number and the distribution of the covariates.

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Randomized Response Technique for Estimating the Population Total

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In this lecture, a new randomized response technique is proposed for sampling surveys. This technique is aimed at protecting respondents' privacy. It is designed the estimating the population total of a quantitative characteristic or the population mean. It provides a high degree of protection to the interviewed individuals; hence it may be favorably perceived and increase readiness to cooperate. Instead of revealing the true value of the characteristic under investigation, the respondent only informs whether the value is larger (or smaller) than a number which is selected by him/her at random and unknown to the interviewer. For each respondent, this number, a sort of an individual threshold, is generated as a pseudorandom number from the uniform distribution. Further, two modifications of the proposed technique are presented. The first modification assumes that the interviewer also knows the generated random number. The second modification deals with the issue that for certain variables, such as income, it may be embarrassing for the respondents to report either high or low values. Thus, depending on the value of the pseudorandom lower bound, the respondent is asked different questions to avoid being embarrassed. The suggested method can be applied to most currently used sampling schemes, including cluster sampling, two-stage sampling, etc. Results of simulations illustrate behavior of the proposed procedure.

Scale-Mixture Extension of Inverse Weibull Distribution

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In this study, scale-mixture extension of inverse Weibull (SIW) distribution is introduced by using a new stochastic representation of the slashing methodology; see Gomez et al. [1] and references therein for further information about the slashing methodology. Some statistical properties of the SIW distribution are derived. Maximum likelihood methodology is used to obtain the estimates of the unknown parameters of the SIW distribution. In the application part, two real data sets from the corresponding literature are modeled via SIW distribution. Also, modeling performance of the SIW distribution is compared with its rivals by means of the well-known information criteria and goodness of fit statistics.

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Positive Solutions for Nonlinear Singular Superlinear Elliptic Equations

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In the past, multiplicity theorems for positive solutions of singular problems were proved by many scholars. The present work and cover a broad class of parametric nonlinear singular Dirichlet problems. So, based on existing research, we consider a nonlinear nonparametric elliptic Dirichlet problem driven by the p-Laplacian and reaction containing a singular term and a (p-1)-superlinear perturbation.Using variational tools together with suitable truncation,comparison techniques, and the critical point theory,we are looking for positive solutions and we prove the existence of at least two positive smooth solutions.

A Dynamic Optimal Control Problem for an Elastic Plate in a Contact with a Rigid Obstacle

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We deal with an optimal control problem governed by a nonlinear hyperbolic initial-boundary value problem describing the perpendicular vibrations of a simply supported anisotropic elastic plate against a rigid obstacle. A variable thickness of a plate plays the role of a control variable. We have considered a viscoelastic case in [1]. The state problem in the elastic case is the initial-boundary value problem for the hyperbolic variational inequality without any damping term. In contrast to a viscoelastic case we obtain accelerations of vibrations in a space of vector measures due to [2]. The state problem is solved through the penalization method. There is no uniqueness of a solution.

In order to obtain an optimal thickness function we restrict the set of states only to deflections received as limits of sequences of penalized solutions. We apply the solutions of control problems corresponding to penalized and regularized state problems i order to derive the approximate necessary optimality conditions.

Acknowledgement. The presented work is supported by the Ministry of Education of Slovak Republic under VEGA Grant 1/0819/17.

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Thickness Optimization of Piecewise Constant Gao Beam

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The contribution deals with an optimal shape design problem of a nonlinear Gao beam subject to a vertical load. The aim of the optimization is to find the thickness distribution to maximize the stiffness of the beam characterized by a compliance cost functional. For modelling the state problem, the nonlinear Gao beam was proposed in 1996 [1]. We consider a stepped beam with piecewise constant thickness distribution. The volume of the beam is preserved and fixed during the optimization. The existence analysis of the optimization problem and convergence properties of discretization using finite element approach is covered. The theory is illustrated by numerical simulations.

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A Polynomial Case of Nonconvex Quadratic Optimization

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We consider the following case of quadratic optimization: given $Q \in \mathbb{R}^{n\times n}$ with rank r and given $p \in \mathbb{R}^n$ and $\underline{x} \leq \overline{x} \in \mathbb{R}^p$ (where inequalities between vectors are understood componentwise), solve $\max_{x\in\mathbb{R}^n} x^TQx + p^Tx$ s.t. $\underline{x} \leq x \leq \overline{x}$. The problem is easily shown to be NP-hard. We show that in the rank deficient case, when $r = O(1)$, the problem becomes polynomially solvable. (Moreover, if Q is positive semidefinite, then our method also yields a polynomially solvable case of the discrete version with integer variables.) The idea is based on a reduction of the problem to the problem of enumeration of faces (of all dimensions) of a certain zonotope in dimension $O(r)$, which can be done by a method by Edelsbrunner et al. [2] in time $n^{O(r)}$. This work extends previous results [1, 3] where Q had been assumed positive semidefinite and $p = 0$, while we can handle an arbitrary Q and p.

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Compositional Cubes and Generalized Linear Models - Similarities and Differences

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Compositional data are commonly known as multivariate observations carrying relative information. The analysis of compositional data arranged in a table, resulting from two underlying factors, has been introduced in the literature as compositional tables approach. This contribution focuses on the extension of that approach on more-factorial compositional data, represented here with three-factorial compositional cubes. The relative nature of compositional data prevents from using standard statistical methods and its real coordinate representation is appropriate prior to the analysis. The structure of the coordinate system proposed for more-factorial compositional data strongly resembles the parameters of linear models. Therefore the standard approach to the analysis of more-factorial data, represented here with the generalized linear mixed effect model, will be compared with the compositional one on a simple example. In particular, differences in the interpretation of results and possibilities of a further inference will be discussed.

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On Higher Order Approximations in the Cox Proportional Hazards Model

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The Cox proportional hazards model is one of the most popular models in survival analysis. The significance of the effect of covariates on time to an event is usually verified by means of the likelihood ratio test, the Wald test, or the score test. These are large sample tests that are only the first order approximations and they do not necessary maintain the significance level for small data sets. Higher order approximations of the likelihood function based on the Barndorff-Nielsen formula [1] and the Lugannani-Rice formula [3] are applied to improve these tests. The lecture is focused on the accuracy of p-value of these tests for small datasets for the Cox model with right-censored and left-truncated observations when only one covariate is considered. The simulations show that higher order approximations based on the Lugannani-Rice and the Barndorff-Nielsen formulas in the combination with the Wald statistic improve the accuracy of p-value [2].

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Bayesian Networks in the Prediction and Diagnostics of Industrial Assembly Lines

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Much of industrial production comes from automatic or semi-automatic assembly lines. In the data-driven age, there are two main challenges concerning assembly lines. [1] Learn the characteristics of the assembly process and predict the quality of the output. [2] Perform diagnostics of the process, i.e. assess how probable various causes are of observed malfunctioning of the line. Both these needs are met by the Bayesian network paradigm. A Bayesian network combines expert knowledge of the process with data obtained during the production. Expert knowledge is used to suggest the structure of the network, and the process data are used to learn the conditional probabilities. The network may be used in the forward mode (i.e. from causes to effects) to predict the quality of the product, or in the backward mode (from effects to causes) for diagnostics. An example of a Bayesian network will be presented which describes an assembly line in the automotive industry.

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Computational Strategies for Subset Regression Model Selection

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Computational strategies for computing the best-subset regression models are proposed. The algorithms are based on a regression tree structure that generates all possible subset models. An efficient branch-and-bound algorithm that finds the best submodels without generating the entire tree is described. Specifically, the computational burden is reduced by pruning the non-optimal subtrees. Strategies and approximate algorithms that improve the computational performance are investigated. Further, this strategies are adapted to solve the problem of regression subset selection under the condition of non-negative coefficients. The solution is based on an alternative approach to quadratic programming that derives the non-negative least squares by solving the normal equations for a number of unrestricted least squares subproblems. This innovative approach is computationally superior to the straightforward method that would estimate the corresponding non-negative least squares of all possible submodels in order to select the best one. The R package "lmSubsets" for regression subset selection is introduced and described. The package aims to provide a versatile tool for subset regression.

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Cross-Correlations in ERP Analysis of Processes Underlying Mental Rotation

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Electroencephalography (EEG) is an noninvasive electrophysiological monitoring method of recording electrical activity of human brain. One of the EEG techniques include event-related evoked potentials (ERPs), often used in cognitive science, which is recording brain response as a direct result of a specific stimulus. The ERPs were obtained as arithmetic average of the signal from repeated trials. The signal were measured by EEG with 61 electrodes from 27 healthy volunteers during an experiment. The experiment consist of visual stimuli included 10 rotated letters displayed in either canonical or mirror-reversed format in random order. The subject's task was to decide whether the stimulus was presented in one of the two formats. Data pre-processing and statistical analyses were performed in the EEGLAB toolbox, MATLAB and R.

Cross-correlation is a measurement of the linear dynamic dependence between the two time series for different values of the lag. We include cross-correlations in brain signal averaging to obtain ERP curves to study processes underlying mental rotation. This method is compared with other approaches of calculating ERPs through Procrustes curve registration or arithmetic averaging using different alignment of the signal. We further discuss the application of cross-correlation in exploration of a (a)symmetry between left and right cerebral hemispheres.

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Analysis of Discretized Models of the Stokes Flow with Local Coulomb's Slip Boundary Conditions

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The Stokes system with local Coulomb's slip conditions is discretized by a mixed finite element method using P1-bubble/P1 elements. Solutions to the discretized problem are defined as fixed-points of an appropriate mapping. We prove the existence of at least one solution, establish conditions under which the solution is unique and analyze how these conditions depend on the discretization parameter and the slip coefficient in Coulomb's law. Numerical experiments will be presented. This is a joint contribution with R.Kučera from VŠB Ostrava and V. Šátek from VUT Brno.

Bivariate Analysis of Compositional Data Using Weighted Symmetric Pivot Coordinates

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Compositional data are multivariate observations with the scale invariance property, i.e. the relevant information is contained in (log)ratios between their components (parts). Therefore, for statistical processing of compositions it is essential to express them first in a proper coordinate system, preferably such a one which aggregates each logratio uniquely in one of the coordinates (Filzmoser et al., 2018). Specifically, if so called pivot coordinates are taken, in the first coordinate (w.l.o.g.) all logratios involving a given part are aggregated. This may not be desirable if data quality problems, e.g., due to measurement error, occur for specific compositional parts, because such parts would contribute through the respective logratios the same weight to the coordinate as parts with good data quality. The influence of poor data quality on the construction of coordinates can be suppressed by constructing coordinates as weighted sums of logratios (Hron et al., 2017). The focus of this contribution is on exploring pairwise associations between compositional parts, following Kynčlová et al. (2017). Coordinates are constructed which cover all relative information of two parts of interest to the remaining parts, with weights applied to the remaining parts representing their data quality. The new coordinate system, called weighted symmetric pivot coordinates, provides a solid ground for correlation analysis with compositional data. The usefulness of the approach will be demonstrated with simulated data and real-world geochemical data.

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Inferring Causal Interactions in Bivariate Time Series

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Identifying causal interactions among observed time series is an important issue in the study of complex systems in diverse scientific fields such as econometrics, neuroscience, and climate science. Granger causality (Granger, 1969) is a very popular technique for assessing the presence of causal relations between time series. It is said that time series X_t Granger causes time series Y_t, if the prediction error of X_t is reduced by including measurements from Y_t in the linear regression model. The application of Granger causality with standard significance testing leads to the detection of spurious causality due to different causes, see e.g. Krakovská et al. (2018), Nolte et al. (2008). Haufe et al. (2013) proposed a procedure based on time-reversed data, called time-reversed Granger causality, to alleviate the problem of spurious causality in the case of independent time series.

Here we test the ability of the time-reversed Granger causality to detect the presence of true interaction between two non-linearly coupled time series. Using simulations, we show that the time-reversed Granger causality does not correctly detect asymmetric causal dependence between two time series in general. Furthermore, we present that the classical Granger causality procedure has higher predictive power to detect absence of causal interaction between time series than the time-reversed Granger causality.

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Parameter Estimation for Stochastic Partial Differential Equations of Second Order

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Consider the following stochastic wave equation with strong damping

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2}(t,\xi) &= b\Delta u(t,\xi) - 2a\frac{\partial u}{\partial t}(t,\xi) + \eta(t,\xi), \quad (t,\xi) \in \mathbb{R}_+ \times D,\\
u(0,\xi) &= u_1(\xi), \quad \xi \in D,\\
\frac{\partial u}{\partial t}(0,\xi) &= u_2(\xi), \quad \xi \in D,\\
u(t,\xi) &= 0, \quad (t,\xi) \in \mathbb{R}_+ \times \partial D,
\end{aligned}$$

where $D \subset \mathbb{R}^d$ is a bounded domain with a smooth boundary, Δ is the Laplace operator, η is a random Gaussian noise and $a > 0$, $b > 0$ are unknown parameters.

We propose strongly consistent estimators of parameters a and b based on observation of the trajectory of the solution $(u(t,\xi),\, 0 \le t \le T,\, \xi \in D)$ up to time T. In the first part, we present estimators based on the norm of the solution and we proceed with estimators based on some "observation window". More specifically, we will be interested in estimators based on observation of the individual coordinates of the solution and we will also discuss asymptotic normality and implementation of these estimators.

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Application of Nonsmooth Optimization in Solving Contact Mechanics Problems

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In the presentation we propose an abstract framework that can be used to numerically approximate a solution to a class of mechanical contact problems. We introduce an abstract nonsmooth optimization problem and prove existence of a unique solution to this problem. Next, we present a numerical scheme approximating the solution and provide numerical error estimation. We apply the abstract theory to a static contact problem describing an elastic body in contact with a foundation. This contact is governed by a nonmonotone friction law with dependence on normal and tangential components of displacement. Weak formulation of introduced contact problem leads to a hemivariational inequality. Finally, we show results of computational simulations and describe the numerical algorithm that is used to obtain these results. This presentation is a joint work with A. Ochal.

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Saturation Overshoot in Unsaturated Porous Media Flow

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Understanding of fluid motion in unsaturated porous media still cannot be considered complete. There are many approaches to modeling porous media flow, the most traditional one being in the framework of continuum mechanics. However, there are several important flow regimes which are not correctly captured by continuum mechanics based modeling. One of the important aspects of continuum mechanics based modeling is the failure to capture the so-called saturation overshoot in gravity-driven fingers. A semi-continuum model for the description saturation overshoot is presented. The model captures qualitatively and quantitatively all features of one dimensional and two dimensional unsaturated porous media flow experiments reported in the literature. The porous medium is described by the retention curve, intrinsic permeability, porosity, and saturation-dependent relative permeability. The fluid is described by its dynamic viscosity and density. The model is based on Macro Modified Invasion Percolation concept of dividing the medium into small blocks, which are not infinitesimal.

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Classification in Analysis of Auditory Evoked Potentials: Functional PCA Framework

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Evoked potentials (EPs) reflect neural processing and are widely used to study sensory perception. However, methods of analyzing EP have been limited mostly to the conventional ensemble averaging of EP response trials to a repeated stimulus, and less so to single-trials analysis. In this study, we applied the functional data analysis approach (FDA) to study auditory EP in the rat model of tinnitus, in which overdoses of salicylate (SS) are known to alter sound perception characteristically, as the same way as in humans. Single-trial auditory EPs were analyzed, after being collected on a daily basis from an awake rat, which had been surgically implanted with intracranial electrodes over its auditory cortex. Single-trial EP integrals were generated with sound stimuli presented systematically over an intensity range. The results were approximated using the cubic spline to give sets of smoothed response-level functions for each of the three sounds. Comparisons between daily intensity-series for each sound type were done using cross-distance measures based on the response-level functions in both the original form and the first-derivative form. From the results of FDA, the first-derivative form was found to provide a clearer separation, when EP data were compared between SS and the Control groups. This is also true when the daily data were compared within the more variable SS-group itself. In addition, at the high intensity region where SS-action is presumably strong, we also observed characteristic changes in two statistical parameters, mean and skewness, of the cross-distance representations. Results suggested that FDA is a sensitive approach for EP studies, and it can become a powerful tool for the research in neural science, particularly neuropharmacology.

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Estimating Mean and Covariance Function in Principal Component Analysis through Conditional Expectation

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Functional data analysis is a relatively novel approach in multivariate statistics which extends the concept of a random variable to the infinite dimensional setting. While the measu-rements of a functional random variable still have finite dimension, it is assumed, that the ob-served variable is in its nature infinitely dimensional [1]. Longitudinal studies are a natural example of data suitable for the functional approach. Multiple variables are measured at several discrete points over time to estimate the underlying continuous function. Due to the sparseness of measured data, conventional techniques for functional data analysis may be unsuitable. To address this issue, a new method was developed – principal component analysis through conditional expectation (PACE). This method heavily relies on a non-parametric estimate of the mean and covariance function [2]. We examined how the quality of these estimates affects the performance of the principal component analysis through conditional expectation, focusing on the choice of the smoothing parameter and its extremes such as over- and undersmoothing.

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Stochastic Optimization Schema with Timers

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We introduce a stochastic optimization schema with random times of changes. The task is to optimize gain from a random process arising in the scheme. The process is controlled by decisions made in random times. We intent to give a description of the schema and present a procedure finding a solution.

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Oscillation of Some Iterated Partial Summations

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The univariate partial summations were defined in general in [3]. If the beginning probability distribution is $\{P_j^*\}_{j=0}^{\infty}$ (called the parent), it is transformed to another distribution $\{P_x^{(1)}\}_{x=0}^{\infty}$ (descendant) with some real function $g(j)$ and a normalizing constant c_1.

When we repeat this transformation with the descendant of the first generation $\{P_x^{(1)}\}_{x=0}^{\infty}$ as the parent of the second generation while function $g(j)$ remains unaltered, with a proper normalizing constant c_2, we obtain as the result another probability distribution - the descendant of the second generation $\{P_x^{(2)}\}_{x=0}^{\infty}$. Similarly the descendant of the n-th generation can be obtained. For $g(j) = c$ the existence of the limit distribution for $n \longrightarrow \infty$ was proved in [4] (the limit distribution is geometric for a wide class of parent distributions). For some types of parental distributions defined on a finite support it is possible to find the limit distribution of such repeated partial summations using the power method (see [1]). An extension to the bivariate case is possible (see [2]).

For some distributions with a finite support for which the conditions of the power method are not satisfied, the limit of the repeated partial summations does not exist and the sequence of descendant distributions oscillates. Some examples will be presented.

Beam in Contact with Deformable Foundation: Convex Case

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The contribution deals with two problems for a nonlinear beam and their numerical solution. These include problems regarding beam bending and contact problems with deformable foundation. The beam model under consideration is the Gao nonlinear model, which was firstly introduced in [1], and problems are restricted only to the stationary problems without friction and with axial force which does not cause buckling.

The approach that was chosen here is based on the so-called control variational method, see [2]. Its authors M. Sofonea and D. Tiba used it to solve contact problem of linear Euler–Bernoulli beam and only for one special case of boundary conditions. The idea of the method is to transform the original problem to an equivalent optimal control problem. Submitted contribution follows papers [3] and [4] which generalizes this method and additionally provides a suitable numerical solution which is illustrated by a number of examples. The algorithms used in these examples were implemented using MATLAB.

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Functional Data Analysis for Probability Density Functions: A Bayes Space Approach

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Modern studies often rely on aggregations of elementary data, which are then represented through probability density functions (PDFs). For instance, in socio-economic contexts, the age of the population is often described through its distribution (i.e., a population pyramid), whereas, in environmental studies, the soil granularity is typically represented through a particle-size distribution. In all these cases the dataset consists of PDFs, whose proper statistical treatment is key to describe, model and predict the phenomenon under study. Statistical methods for the analysis of PDFs need to account for the infinite-dimensionality of the data, and their inherent constraints. We will discuss the Bayes space viewpoint to the analysis of PDFs, which combine the approaches of functional data analysis and compositional data analysis, through the foundational role of the generalized Aitchison geometry. In this framework, methods for dimensionality reduction [1], modeling and prediction [2,3] will be illustrated, with application to studies of industrial and environmental interest.

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Identification of Mineralization in Geochemistry Based on the Spatial Curvature of Log-Ratios

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Detecting subcropping mineralizations but also deeply buried mineralizations is one important goal in geochemical exploration. The identification of useful indicators for mineralization is a difficult task as mineralization might be influenced by many factors, such as location, investigated media, depth, etc. We propose a statistical method which indicates chemical elements related to mineralization. The identification is based on GAM models for the element concentrations across the spatial coordinate(s). The log-ratios of the GAM fits are taken to compute the curvature, where high curvature is supposed to indicate mineralization. By defining a measure for the quantification of high curvature, the log-ratios can be ranked, and elements can be identified that are indicative of the anomaly patterns.

In the presentation we will show results from our data set, and also results from other geochemical data sets. All these results indicate that the method indeed is able to identify pathfinder elements for mineralization. Note that our proposed approach in unsupervised – thus it is not necessary to know the locations of the potential mineralization.

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The Halfspace Depth Characterization Problem

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The halfspace (or Tukey) depth is an inferential tool that aims to generalize quantiles to multivariate datasets. It has been long conjectured that, just as for the usual quantiles, there is a one-to-one relation between all Borel probability measures, and all possible depth surfaces. We answer this conjecture in the negative. That suggests an interesting open problem of characterizing those probability measures that possess a unique depth. A complete solution to this problem would have far-reaching implications, not only in the theory of multivariate statistics.

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Rank Theory Approach to Ridge, LASSO, Preliminary Test and Stein-Type Estimators

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In the development of efficient predictive models, the key is to identify suitable predictors for a given linear model. This contribution provides a comparative study of ridge regression, LASSO, preliminary test and Stein-type estimators based on the theory of rank statistics. Under the orthonormal design matrix of a given linear model, we find that the rank based ridge estimator outperforms the usual rank estimator, restricted R-estimator, rank-based LASSO, preliminary test and Stein-type R-estimators uniformly. On the other hand, neither LASSO nor the usual R-estimator, preliminary test and Stein-type R-estimators outperform the other. The region of domination of LASSO over all the R-estimators (except the ridge R-estimator) is the interval around the origin of the parameter space. Finally, we observe that the L_2-risk of the restricted R-estimator equals the lower bound on the L_2-risk of LASSO.

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Buckling Analysis of Nonlinear Gao Beam

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In a mathematical sense, buckling is a bifurcation in the solution to the equations of static equilibrium. It is well known that if a beam is subjected to an axial compressive load, the total potential energy of the beam becomes nonconvex as soon as this load reaches certain limit value. The classical Euler–Bernoulli beam is still the most popular beam model. However, this linear model can be used only for infinitesimal deformation and does not allow a regular solution to the buckling problem. Euler proposed in 1757 a solution using eigenvalues and eigenfunctions, which is still widely used, but it has qualitative character only.

Nonlinear mathematical model developed by D.Y. Gao in [1] is intended for bending problems with moderately large deformation and for buckling analysis. It is more appropriate for this purpose but its solution is not easy task. Several attempts have already been made by D.Y. Gao who used his canonical dual finite element method. We present here a formulation based on the control variational method (see e.g.[2]) which idea consists in a transformation of the original problem into an optimal control problem. Such a solution was successfully used in papers [3] and [4] for convex cases, which of course does not include buckling. We provide an analysis of the transformed problems particularly with regard to possible numerical solution.

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System Failure Estimation Based on Field Data and Semi-Parametric Modeling

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A top-priority task nowadays is to ensure quality, safety, and dependability of technical systems. One of the ways to avoid failures is by monitoring the conditions and degradation of the system using diagnostic signals.

The analyzed dataset covers a relevant set of field data of monitored vehicle fleet and has a form similar to sparse longitudinal data. Our approach to the analysis is based on semi-parametric modeling. In particular, generalized additive models are applied to obtain the appropriate description of the mean and variability of the analyzed field data. Further, stochastic differential equation for time-dependent Ornstein-Uhlenbeck process with estimated drift and diffusion term is considered. Numerical methods are used to generate trajectories of such random process and to obtain a sample of first hitting times. These are used to infer on the survival function and failure rate and to predict the mean residual useful life.

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Statistical View on the Real Estate Market in the Czech Republic

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The aim of this contribution is to make basic statistical analyzes of apartment prices in various municipalities of the Czech Republic in the period 07/2018 – 05/2019. The real estate market is not subject to frequent analyzes in the academic field, which is due to the difficulty in obtaining data. It can be said, therefore, that freely available and published data do not provide the possibility to perform more comprehensive statistical analyzes or even more detailed regional analyzes. We will work with data obtained through the parsing of large real estate server websites. By automatic polling, we are able to get data on the floor area of advertised apartments and the asked purchase price. We will present basic statistical characteristics in different months of the monitored period and in different regions of the Czech Republic. Special attention will be paid to the analysis of the impact of the tightening of mortgage rules and the impact of interest rates on average housing prices. Using cluster analysis, according to the same evolution of housing prices, we try to divide the districts in the Czech Republic. Special attention will be paid to modelling the relationship of apartment prices between neighbouring small municipalities. A spatial lag model will be used for the calculation. We will assemble a model for describing the relationships between housing prices and suitable explained variables. To using this model, we are inspired by studies from [Kotatkova, 2015] and [Bhatacharjee],

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Solution of Inverse Problem for Gao Beam Using Optimal Control Problem

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In this contribution we deal with inverse problem for nonlinear Gao beam model which was firstly introduced by D. Y. Gao in 1996. As an inverse problem the identification of unknown material parameters for the nonlinear Gao beam is meant. The inverse problem will be formulated as the optimal control problem which consists of minimization of cost functional and state problem. The existence of the unique solution will be presented.

The numerical realization of the problem is based on using finite elements method, thus the discretization of the state problem and functional has to be done. The minimization of the discretized functional is based on gradient method with a suitable step size calculations. The gradients of the discretized functional are computed by using the adjoint problem technique. Numerical results for the nonlinear Gao beam model are compared with results for the classical linear Euler-Bernoulli beam model and numerical computations are realized by using mathematical software MATLAB.

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Compositional Analysis of Untargeted Metabolomic Data Using Multiple Bayesian Hypotheses Testing

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Currently, both clinical targeted and untargeted metabolomic approaches aim to find statistically significant differences in chemical fingerprints of patients with some disease and a control group and to identify biological markers allowing a prediction of the disease. Traditionally, the differences between controls and patients are evaluated by both univariate and multivariate statistical methods. The univariate approach relies merely on t-tests (or their nonparametric version) where the results from multiple testing are compared by p-values and fold-changes using a so-called volcano plot. As a counterpart, a multiple Bayesian hypotheses testing is proposed, introducing a concept of b-values as well as a Bayesian version of the volcano plot incorporating distance levels of the posterior highest density intervals from zero. Moreover, since each metabolome is a collection of some small-molecule metabolites in a biological material, relative structure of metabolomic data is of the main interest. A proper choice of orthonormal coordinates w.r.t. Aitchison geometry considering the compositional character of a metabolome is, therefore, an essential step in any statistical analysis of such data. The theoretical background is accompanied by an analysis of a data set containing plasma of patients suffering from an inherited metabolic disorder of ketone body synthesis and leucine degradation – 3-hydroxy-3-methylglutaryl-coA lyase deficiency (HMGCLD).

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Spatio-Temporal Adaptive Penalised Spline Models

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Penalised regression splines (P-splines, [1]) models have achieved great popularity both in statistical and in applied research. From a statistical point of view, the reason for their popularity is their applicability to different fields, from survival analysis and spatial and spa-tio-temporal statistics to functional data analysis. From the applied point of view, the metho-dological developments have allowed analysing and understanding complex biological and health phenomena. A possible drawback of P-spline models is that they assume a smooth transition of the covariate effect across its whole domain. However, in some practical applications more complex situations arise, with effects that may not change in some regions of the covariate, while changing rapidly in other regions. In these situations, it is desirable and needed to adapt smoothness locally to the data, and adaptive P-splines have been suggested (e.g., [2]). However, the extra flexibility afforded by adaptive P-splines is obtained at the cost of a very high computational burden, especially in a multidimensional setting (e.g., two-dimensional interaction surfaces). Furthermore, to the best of our knowledge, the literature lacks proposals for adaptive P-splines in more than two dimensions. Motivated by the need of analysing data derived from experiments conducted to study neurons' activity in the visual cortex of behaving monkeys, this work presents a locally adaptive P-spline model in three dimensions (space and time). Estimation is based on the SOP (*Separation of Overlapping Precision Matrices*) method [3], which provides the stability and speed we look for. This is joint work with Maria Durban, Dae-Jin Lee, Paul H.C. Eilers and Francisco Gonzalez.

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The Parallel Factor Analysis and the Tucker Model: A Simulation Study

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The electroencephalogram (EEG) provides a useful tool for the description of the neural activity of a brain. Analysis of multichannel, spatially distributed, EEG information is preferred over a separate analysis of EEG signal from each electrode. However, because of the multiway character of the EEG data, the hidden sources of neural activity cannot be reliably extracted by standard statistical methods like the principal component analysis (PCA) or factor analysis.

The parallel factor analysis (PARAFAC) [1, 2] represents a generalisation of PCA in higher dimensions and can be used for detection of hidden factors of multichannel EEG in time, space and frequency domain [3]. However, when the measurement with a lower number of EEG electrodes is analyzed, the resemblance of the observed spatial distribution of several, in frequency not overlapped oscillatory sources, may indicate that the usage of a more flexible Tucker3 model can be preferred.

In this study, we validate and compare the PARAFAC and Tucker3 model on two types of simulated EEG data; i) with a different spatial location of neural activity of the target frequencies and ii) with a different number of electrodes distributed over the whole scalp. The aim of this analysis is to better understand situations where the Tucker3 model leads to a more parsimonious representation of EEG data, but with a comparable explanation of neural activity variability as the PARAFAC model.

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On Generalizations of the Log-Linear Model for Multivariate Categorical Data

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Multivariate categorical data are most often analyzed by fitting log-linear models [4]. The talk describes the class of relational models [3], which are more general, in that the sample space does not have to be a Cartesian product, the effects are not necessarily associated with cylinder sets and the overall effect may or may not be present. After discussing examples which motivate these generalizations, the talk will consider estimation and testing in relational models. When the overall effect is not present, the usual equivalence of the maximum likelihood estimates under multinomial and Poisson sampling does not hold. In the multinomial case, the maximum likelihod estimates reproduce the observed subset sums only to a constant of proportionality, and in the Poisson case they do not reproduce the observed total. When the data also contain zeros, depending on their pattern, the MLE may only exist in the closure of the original model with respect to the Bregman divergence, which coincides with the set of pointwise limits of the distributions in the relational model [1]. Relational models without the overall effect are curved exponential families, and the statistical meaning of adding or removing the overall effect will be described by concepts from algberaic geometry [2].

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A Permutation Approach to the Analysis of Spatio-Temporal Geochemical Data in the Presence of Heteroscedasticity

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In the framework of functional data analysis, the paper introduced a permutation-based test for the effect of covariates for a space-time regression model in the case of heteroscedasticity. To test for the effect of covariates, the permutation of estimated residuals of the spatial regression model instead of the observations themselves is proposed. A weighted least-squares model is fitted to the observations, leading to approximately exchangeable, and thus permutable, residuals. The performance of the test is assessed through a simulation study and compared to the ordinary least-squares approach. The proposed test perfoms similarly to the ordinary least-squares approach for equal sample sizes and better for the unbalanced design, where neglecting the heteroscedasticity biases the size of the ordinary least-squares test. The methodology is demonstrated on a real-world geochemical data set. Three different spatio-temporal models are used to analyse behaviour of potassium chloride pH, water pH and percentages of organic carbon data in a soil at the border between the forest and field.

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R and Matlab Tools for Kernel Estimation and Visualization of Hazard Function in Survival Analysis

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The hazard function is a useful tool in survival analysis and reflects the instantaneous probability that an individual will die within the next time instant. In practice, the hazard function may be affected by other variables.

The aim of the contribution is to present developed software tools for estimation of the unconditional and conditional hazard function based on kernel smoothing techniques. The value added of these new packages is the graphical presentation of the relationship of survival time and some continuous characteristic.

The R package **kernhaz** is available from the official archive CRAN. The alternative package for Matlab users can be downloaded from [2]. These packages implement estimates of hazard function for right-censored data and include also two bandwidth selection methods.

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On the Optimal Control of Variational-Hemivariational Inequalities

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Variational and hemivariational inequalities are widely used in the study of many nonlinear boundary value problems and have a large number of applications in Contact Mechanics and Engineering. The theory of variational inequalities was developed in early sixty's, by using arguments of monotonicity and convexity, including properties of the subdifferential of a convex function. In contrast, the analysis of hemivariational inequalities uses as main ingredient the properties of the subdifferential in the sense of Clarke, defined for locally Lipschitz functions, which may be nonconvex. Variational-hemivariational inequalities represent a special class of inequalities, in which both convex and nonconvex functions are present. Recent references in the field include the book [1]. Optimal control problems for variational and hemivariational inequalities have been discussed in several works, including [2].

We start this lecture with some notation and preliminary results. Then, we introduce a variational-hemivariational inequality in which all the data depend on a parameter p. We state the behavior of the solution of this inequality with respect to p and provide a convergence result. Next, we consider a class of optimal control problems associated to the variational-hemivariational inequality, for which we prove the existence and convergence of the optimal pairs.

The mathematical tools developed in this lecture are useful in the analysis and control of a large class of boundary value problems which, in a weak formulation, lead to elliptic variational-hemivariational inequalities. To provide an example, we illustrate our results in the study of an inequality which describes the equilibrium of an elastic body in frictional contact with a foundation made of a rigid body covered by a layer of soft material.

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Conic Optimization in Geotechnical Stability Analysis

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This contribution is motivated by solution of stability problems like stability of slopes, foundations or tunnel. The aim is to find a safety factor of a structure subject to applied loads. This factor may be defined by parametrization of the load and consequent increasing of this parameter up to its limit value. Beyond the limit value, the structure collapses. If the structure obeys an associative and perfectly plastic law then the limit load factor may be defined directly (without any incremental procedure) by a specific convex optimization problem, the so-called limit analysis problem. This problem can be formulated either in terms of stresses (the static approach) or in terms of displacement rates (kinematic approach). Both the approaches are in mutual duality. For classical yield criteria, the limit analysis problem contains conic constraints defined in each material point. From a solution of this problem, one can also predict failure zones describing the collapse of the structure.

In this contribution, we describe two innovative ideas enabling a deeper understanding of this problem. First, we introduce a specific inf-sup condition defined on the related conic contraint set and show that its validity is important for both theoretical and numerical analysis. In particular, one can prove the equivalence between the static and kinematic approaches and derive the analytical upper bounds of the limit load using functions which need not belong to the restrictive conic set. These bounds may be computable if the inf-sup constant is known.

Second, we introduce a penalization method for solution of the kinematic problem to remove the conic constraints. The penalized problem may be solved by standard finite elements and Newton-like methods due to available convergence analysis. To achieve more accurate results, we complete this numerical strategy with continuation over the penalty parameter and with local mesh adaptivity. For each numerical approximation of the unknown kinematic field, one can easily compute the guaranteed upper bound mentioned above. It leads to a posteriori information about numerical errors.

We illustrate the efficiency of our methods on numerical examples including slope stability or strip-footing benchmarks. Further, we present that the methods are convenient even for strongly heterogeneous composite materials. The contribution is a joint work with prof. J. Haslinger (Prague) and prof. S. Repin (Jyvaskyla and St. Petersburg).

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Comparism of Different Statistical Models Used in Shape Index Calculation

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Spatial interpolation and smoothing is usually done for one surface. In our case, we have random samples of such surfaces represented by human faces. Human faces, captured by stereo-photogrammetry are characterised by about 150,000 points. These points are triangulated by about 300,000 triangles. The number of points is extremely high for the purpose of statistical analyses, therefore the 3-dimmensional (3D) coordinates of landmarks and semi-landmarks on curves or surface patches sufficiently characterising the shape have to be automatically identified and this simplified model comprising about 1000 points is then used in further statistical modelling (see Bowman et al. 2015 and Vittert et al. 2019). The identification of (semi)landmarks is a complex process during which B-splines, P-splines and thin-plate splines are used together with differential geometry characteristics of a face, including principal curvatures and shape index (see Katina et al. 2016 and Vittert et al. 20 19).

Shape index, the measure of local surface topology, is calculated using several different linear statistical models of z coordinates on x and y coordinates, i.e. quadratic with interaction without and with intercept, cubic with interaction of x and y without and with intercept (with and without other interactions), and similar models of higher order. The estimates of regression coefficients related to the quadratic terms and their interaction are elements of Weingarten matrix from which the principal curvatures are calculated. These models are applied on sufficiently large neighbourhood of all 150,000 points in local 3D coordinate system. The results are compared numerically and visually by static images of the human face in different views, i.e. frontal, lateral and vertical, and by animations. All statistical analyses and visualisations are performed in R.

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Weighted Pivot Coordinates in PLS Regression with Compositional Covariates and Its Application to Metabolomic Data

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Common types of metabolomic data meet properties of compositional data, i.e. multivariate observations with positive parts (representing metabolite abundances) carrying relative information. Prior to applying standard statistical methods, it is advantageous to map compositional data from their sample space equipped with the Aitchison geometry into standard Euclidean real space using a proper logratio coordinate system [4]. The so-called pivot coordinates [1] are useful for this purpose as they are orthonormal and the first coordinate captures all the relative information about a part of interest in terms of (scaled) sum of all logratios with that part. On the other hand, because logratios which are aggregated into the first pivot coordinate may represent completely different processes, in many cases its weighted counterpart [2] seems to be preferable. A concrete case is represented by regression of a real response on compositional covariates which aims at revealing significant signals in metabolomic spectral data. For this purpose, partial least squares (PLS) regression is used which models relationships between a response and a large set of explanatory variables by means of latent factors. PLS regression has been also widely used to model chemical or biological outcomes from high-dimensional metabolomic profiles [3]. This contribution demonstrates the benefits of using weighted pivot coordinates in PLS regression with metabolomic data in order to enhance the identification of the most relevant spectral signals. For this purpose, the weights are determined according to correlations of pairwise explanatory logratios representing the spectral data with the response variable.

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Weighted Bayes Spaces

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Probability density functions (PDFs) can be understood as functional data carrying relative information. As such, standard statistical methods proposed by functional data analysis (FDA) [3] are not appropriate for their statistical processing. They are typically designed in L2 space (with Lebesgue reference measure), thus cannot be directly applied to densities, as the metrics of L2 does not honor their geometric properties. This has recently motivated the construction of the so-called Bayes Hilbert spaces [3], which result from the generalization to the infinite dimensional setting of the Aitchison geometry for compositional data. More precisely, if we focus on PDFs restricted to a bounded support $I \subset R$ (that is mostly used in practical applications), they can be can be represented with respect to Lebesgue reference measure using the Bayes space of positive real functions with square-integrable logarithm. The reference measure can be easily changed through the well-known chain rule and interpreted as a weighting technique in Bayes spaces. Moreover, it impacts on the geometry of the Bayes spaces which results in so-called weighted Bayes spaces. The aim of this contribution is show the effects of changing the reference from Lebesgue measure to a general probability measure focusing on its practical implications for the Simplicial Functional Principal Component Analysis (SFPCA) [2]. A centered log-ratio transformation is proposed to map a weighted Bayes spaces into an unweighted L2 space (i.e. with Lebesgue reference measure), thus it enable us to apply standard statistical methods such as SFPCA on PDFs.

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Depth-Based Classification: Modified k-Nearest Neighbour Method

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In the current paper we recall the notion of data depth and how it can be used for classification. Then we show how the classical k-nearest neighbour (kNN) method can be modified using data depth. The idea is quite simple – the nearest neighbours of a given point are the points in the training set with similar depth w.r.t. empirical distribution corresponding to a given class (based on the training set) as the given point. We consider the two class classification problem. To classify a new observation, the modified kNN procedure first finds its k-nearest neighbours w.r.t. one class and then its k-nearest neighbours w.r.t. the second class. These points define distributional neighbourhoods. The classification is based on comparison of the volumes of these neighbourhoods. We studied the performance of the newly suggested procedure. In the contribution, we also outline some practical problems that need to be solved when implementing the procedure.

Compressible Nonlinearly Viscous Fluids: Asymptotic Analysis

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Concerning three-dimensional mathematical models of fluids, an analytical solution is often impossible to derive and numerical solution can be unduly complicated. Thus, we need to simplify three-dimensional models, when possible, prior to solving the problem. We show how to derive rigorously lower dimensional models for Navier-Stokes equations for compressible nonlinearly viscous fluids.

Cellwise Robust Regularized Discriminant Analysis

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Quadratic and Linear Discriminant Analysis (QDA/LDA) are the most often applied classification rules under normality. In QDA, a separate covariance matrix is estimated for each group. If there are more variables than observations in the groups, the usual estimates are singular and cannot be used anymore. Assuming homoscedasticity, as in LDA, reduces the number of parameters to estimate. This rather strong assumption is however rarely verified in practice. Regularized discriminant techniques that are computable in high-dimension and cover the path between the two extremes QDA and LDA have been proposed in the literature. However, these procedures rely on sample covariance matrices. As such, they become inappropriate in presence of cellwise outliers, a type of outliers that is very likely to occur in high-dimensional datasets. We propose cellwise robust counterparts of these regularized discriminant techniques by inserting cellwise robust covariance matrices. Our methodology results in a family of discriminant methods that (i) are robust against outlying cells, (ii) provide, as a by-product, a way to detect outliers, (iii) cover the path between LDA and QDA, and (iv) are computable in high-dimension. The good performance of the new methods is illustrated through simulated and real data examples.

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History-Dependent Sweeping Processes: Well-Posedness and Applications

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This work is devoted to the study of a class of sweeping processes with history-dependent operators. A well-posedness result is obtained, including the existence, uniqueness, and stability of the solution. Our approach is based on the variable time step-length discrete approximation method combined with a fixed point principle for history-dependent operators. Then, a quasi-static frictional contact problem for viscoelastic materials with unilateral constraints in velocity is considered. The abstract result is applied in the study of this problem in order to provide its unique weak solvability as well as the continuous dependence of the solution with respect to the initial data.

Practical Aspects of the Vehicle Routing Problem

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One of the common practical applications of operational research is vehicle routing, where the goal is to find an efficient route of transportation of goods to customers through a complex network. In this contribution, we deal with the description and solution of a practical routing problem that can be considered a generalization of the known Capacitated Vehicle Routing Problem (CVRP), see [1]. The main difference from the classical VRP is that the optimization must be performed for a multi-day period and the individual orders can be delivered to the customer a few days in advance. In addition, the number of vehicles available in one day is limited, the vehicle capacity is limited, and the route length is also limited. The goal is to assign the transport of orders into particular days and particular vehicles so that the daily number of vehicles exceeds the prescribed maximum as little as possible, so that no vehicle is overloaded, and so that the total route length is minimized. The solution of the problem is divided into two phases. In the first phase, orders are assigned to routes using a custom modification of the well-known Clarke-Wright Algorithm (see [2]). In the second phase, routes are assigned to individual days.

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