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INFORMATION MEASURE FOR VAGUE SYMBOLS

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Classical (probabilistic) information measure

Information source (probabilistic)

$$(A, p)$$

A – alphabet, $a \in A$ – *symbol*,

p – probability distribution

Information, transmitted by symbol $a \in A$ is

$$I_p(a) = \log_2(1/p(a)) = -\log_2(p(a)) .$$

C. Shannon and V. Weaver – 1948.

Probabilistic Entropy

- uncertainty characteristics of the entire source

Entropy (probabilistic) of the information source *(A, p)* is the mean value of the information measures $I_p(a)$.

$$H(A,p) = \sum_{a \in A} p(a).\ I_p(a)$$
$$= -\sum_{a \in A} p(a).\ \log_2(p(a))$$

Note the „direction“ of the concepts development:

From information of individual symbol to the „information“ of entire source

Fuzzy Source

The Fuzzy information source is a pair

$$(A, \mu)$$

A – alphabet, *μ* – membership function over *A* .

Note that **fuzzy information source is a fuzzy subset of the alphabet**.

Hence – the **information measure of the entire fuzzy source** can be interpreted as a **measure of „fuzziness“** of a fuzzy set.

Naturally, the uncertainty of the fuzzy source (its fuzzy entropy) is in the focus of interest

Several attempts to measure the **fuzzy entropy**.

- A. De Luca and S. Termini (1972)

$$H_{LT}(A,\mu) = -K.\sum_{a\in A} \mu(a).\ \log_2(\mu(a))\ .$$

- A. Kolesárová and D. Vivona (2001)

$$H_{KV}(A,\mu) = -K.\sum_{a\in A} [\ \mu(a).\ \log_2(\mu(a)) + (1-\mu(a)).\ \log_2(1-\mu(a))\ ]\ ,$$

where K is a positive constant.

Fuzzy information of particular symbol

The „individual“ information transmitted by particular symbols is **not explicitely defined**.

But, $H_{LT}(A,\mu)$ and $H_{KV}(A,\mu)$ implicitely include the pattern used in the probabilistic model.

$$I_{\mu}(a) = -\log_2(\mu(a)),\ a \in A$$

(with some formal modifications).

This approach is correct but not typically „fuzzy-like“:

- *This fuzzy information is additive and not monotonous.*
- *The logarithm is a „bribery“ to probabilistic patterns.*

What about some alternative approach ?

GENERAL PROPERTIES OF ANY FUZZY MEASURE OF INFORMATION

$$I_{\mu}(a)\ ,\ a \in A.$$

1) if $a, b \in A$, $\mu(a) \geq \mu(b)$, then $I_{\mu}(b) \geq I_{\mu}(a)$,
2) $I_{\mu}(a) \in [0, 1]$,
3) $I_{\mu}(a) = 0 \Leftrightarrow \mu(a) = 1$.

Conditions (1), (2), (3) are general enough

- **Proposition.** **Fuzzy information measure**

$$I_{\mu}(a) = -\log_2(\mu(a))$$

fulfils conditions (1) and (3), and $I_{\mu}(a) \geq 0$. If the alphabet A is finite then there exists K such that (2) is fulfilled, as well.

An alternative concept of fuzzy information

The mapping $I_m : A \to R$ such that

$$I_m(a) = 1 - \mu(a)$$

Is called **monotonous fuzzy information**

Proposition. Monotonous fuzzy information measure fulfils conditions (1), (2) and (3).

Let us note that it is monotous in the sense of fuzzy set theoretical paradigma.

Extension of fuzzy information - generally

If we wish to extend an information measure $I(\,.\,)$ from A to A^n, we would respect a general condition. For any $\mathbf{a}=(a_1, a_2, \ldots, a_n) \in A^n$, we demand

(4) $I(\mathbf{a}) \geq \max(I(a_1), I(a_2), \ldots, I(a_n))$.

Proposition. Condition (4) is fulfilled also for the probabilistic information measure

$$I_p(a) = \log_2(1/p(a)) = -\log_2(p(a))$$

Extension of monotonous fuzzy information

Let us extend the information measure $I_m(\,.\,)$ from A to A^n.

First, we extend μ .

Let $\mathbf{a} = (a_1, a_2, \ldots, a_n) \in A^n$. ***Then we put***

$$\mu(\mathbf{a}) = \min(\mu(a_1), \mu(a_2), \ldots, \mu(a_n)),$$

and

$$I_m(\mathbf{a}) = 1 - \mu(\mathbf{a}) .$$

The above extension is consistent with the general condition

Proposition. **The extension of monotonous fuzzy information $I_m(\mathbf{a}) = 1 - \mu(\mathbf{a})$ fulfils condition (4).**

Proposition. **Also the fuzzy information $I_\mu(a) = -\log_2(\mu(a))$ fulfils condition (4) if we extend the membership function $\mu(a)$ due to**

$$\mu(\mathbf{a}) = \min(\mu(a_1), \mu(a_2), \ldots, \mu(a_n))$$

☺

And it is all

Thanks for your patience