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A Hybrid Procedure for Network Multi-Criteria Systems

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Outline

- Network systems
- DEMATEL
- ANP
- PROMETHEE
- Combination of methods
- Conclusions

Network systems

Many of today's systems are characterized by

- a network structure and
- evaluation of alternatives by multiple criteria.

Network systems contain both positive and negative feedbacks.

Network system: elements, relations

- Elements: agents, criteria, resources, alternatives, ...
- Relations: influence linkages

Network system represented by a graph

- Elements – nodes
- Relations – edges

DEMATEL (Decision Making Trial and Evaluation Laboratory)

- **Step 1.** *Find the initial direct relation matrix.*

n elements, *m* experts, Each expert makes pairwise comparisons between any two elements by score ranging from 0, 1, 2, 3, and 4. We can then compute the (*n*, *n*) average matrix **A** for all expert opinions.

- **Step 2.** *Calculate the normalized initial direct-relation matrix*.

The normalized initial direct-relation matrix **D**

$$\mathbf{D} = \frac{1}{s}\mathbf{A} \quad \text{, where} \quad s = \max\left(\max_{1 \le i \le n} \sum_{j=1}^{n} a_{ij}, \max_{1 \le j \le n} \sum_{i=1}^{n} a_{ij}\right)$$

Note that each element of matrix **D** is between zero and one.

DEMATEL (cont.)

- **Step 3.** *Compute the total relation matrix.*

$$\mathbf{T} = \mathbf{D} + \mathbf{D}^2 + \ldots + \mathbf{D}^k =$$
$$\mathbf{D}(\mathbf{I} + \mathbf{D} + \mathbf{D}^2 + \cdots + \mathbf{D}^{k-1}) = \mathbf{D}(\mathbf{I} - \mathbf{D})^{-1}, \text{as } k \to \infty.$$

r_i be the sum of i-th row in matrix **T**. Then r_i shows the total effects, both direct and indirect, given by element i to the other elements

c_j be the sum of j-th column in matrix **T**. Then c_j shows the total effects, both direct and indirect, received by element j from the other elements.

- **Step 4.** *Set a threshold value and obtain the impact-relations-map.*

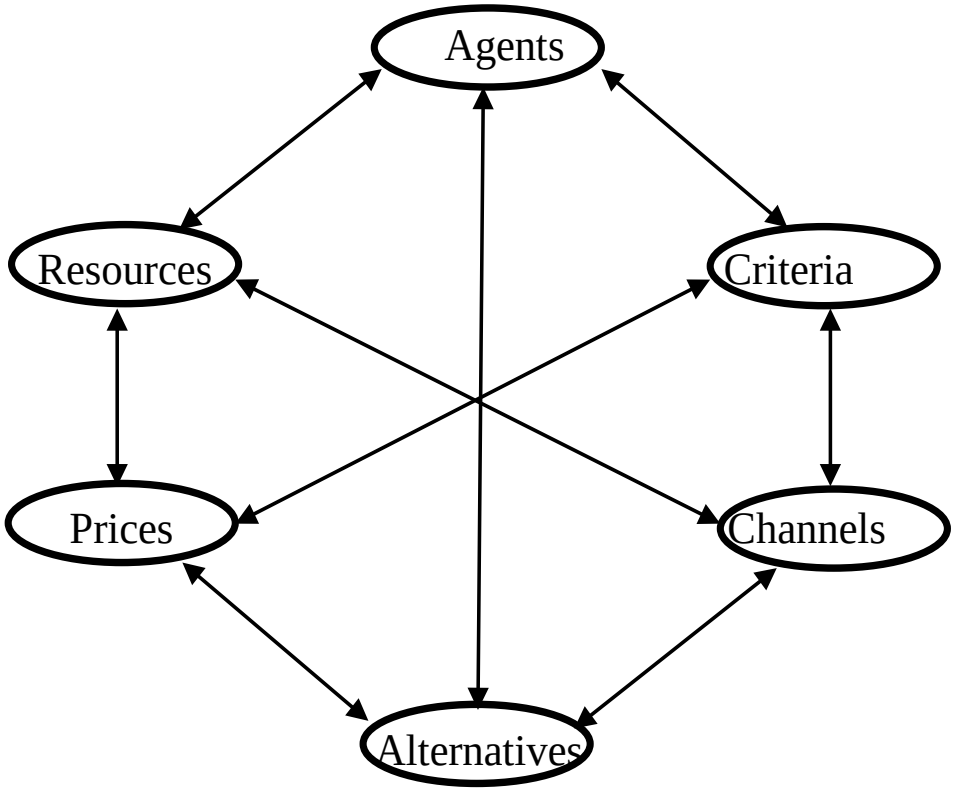
It is necessary to set a threshold value p to filter out some negligible effects in matrix **T**.

Only some elements, whose effect in matrix **T** is greater than the threshold value, should be chosen and shown in an impact-relations-map (IRM).

ANP (Analytic Network Process)

- The Analytic Network Process (ANP) is the method that makes it possible to deal systematically with all kinds of dependence and feedback.
- The well-known AHP theory is a special case of the Analytic Network Process that can be very useful for incorporating linkages.
- The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements.

Clusters and connections



Setting priorities by ANP

- **Step 1**. ***Supermatrix***

  A supermatrix is a matrix of all elements by all elements. Paired comparisons are needed for all the connections in the model. The ANP derives ratio scale priorities by making paired comparisons of elements by using a 1 to 9 scale of absolute numbers.

  The weights from the paired comparisons are placed in the appropriate column of the supermatrix. The sum of each column corresponds to the number of comparison sets.

- **Step 2.** ***Weighted supermatrix***

  The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. Each column of the weighted supermatrix sums to one and the matrix is column stochastic.

- **Step 3.** ***Limited supermatrix***

  Powers of weighted supermatrix can stabilise after some iterations to limited supermatrix. The columns of each block of the matrix are identical and we can read off the overall priority

PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations)

- **Step 1.** ***Formulation of a criteria matrix and weights***

The multicriteria evaluation problem is defined by set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$ and set of evaluation criteria $F = \{f_1, f_2, \ldots, f_k\}$. The evaluations of alternatives by criteria can be expressed in a criteria matrix. The importance of criteria can be expressed by a weight vector.

- **Step 2:** ***Calculation of multicriteria preference indices***

preference functions $P: A \times A \rightarrow [0,1]$.

Preference function values depend on the difference d evaluation of alternatives according to the criterion f

$P(a_i, a_j) = P(f(a_i) - f(a_j)) = P(d)$.

To express preferences in both directions it is possible to define a function

$H(d) = P(a_i, a_j)$, $d \geq 0$,

$H(d) = P(a_j, a_i)$, $d \leq 0$.

Generalized criteria

1. Usual criterion

$$H(d) = 0\ , \quad d = 0, \qquad H(d) = 1\ , \quad d \neq 0.$$

2. Quasi-criterion

$$H(d) = 0\ , \quad |d| \leq q, \qquad H(d) = 1\ , \quad |d| \geq q\ .$$

3. Criterion with linear preference

$$H(d) = d/p\ , \quad |d| \leq p, \qquad H(d) = 1, \quad |d| \geq p\ ,$$

4. Level criterion

$$H(d) = 0\ , \ |d| \leq q\ , \qquad H(d) = 1/2, \ q < |d| \leq p\ , \quad H(d) = 1, \ |d| > p$$

5. Criterion with linear preference and indifference area

$$H(d) = 0\ , \ |d| \leq q\ , \quad H(d) = (|d| - q)\ /\ (p - q)\ , \ q < |d| \leq p\ , \quad H(d) = 1, \ |d| > p$$

6. Gaussian criterion

$$H(d) = 1 - \exp\ (-\ d^2/s^2)\ .$$

PROMETHEE (cont.)

Preference function values $P_h(a_i, a_j)$.

Multicriteria preference index $\pi(a_i, a_j) = \sum_{h=1}^{k} w_h P_h(a_i, a_j)$

- **Step 4:** ***Calculation of outranking flows***

The positive outranking flow is of the form: $F^+(a_i) = \sum_{a_j \in A} \pi(a_i, a_j)$

The negative outranking flow is of the form: $F^-(a_i) = \sum_{a_j \in A} \pi(a_j, a_i)$

The net outranking flow is applied and is of the form:

$F(a_i) = F^+(a_i) - F^-(a_i)$.

Advantages of the methods

DEMATEL

- Expert evaluations of relations in network system
- Threshold value – filter of some parts in the model - reduction

ANP

- Clusters
- All kinds of dependence and feedback
- More precise evaluation of relations

PROMETHEE

- Preference functions
- Calculation of outranking flows between alternatives

Combination of the methods

- **Step 1. DEMATEL - to clarify relations of elements in the network system**
- **Step 2. ANP - to form an unweighted supermatrix by pairwise comparisons.**
- **Step 3. The weighted supermatrix is obtained by multiplying the total-influence matrix (DEMATEL) with unweighted supermatrix (ANP) method.**
- **Step 4. ANP – to get the limited supermatrix with weighs**
- **Step 5. PROMETHEE – evaluation of flows between alternatives with weights from the limited supermatrix (ANP)**

References

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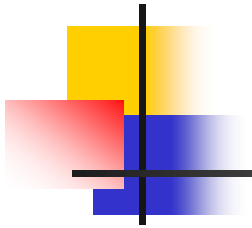
Conclusions

- Network systems are very popular.
- Preference elicitation is the key feature for network systems - a complex problem.
- DEMATEL is easy method for initial evaluation of relations
- ANP is an appropriate approach for preference elicitation in networks
- PROMETHEE – detailed evaluation of alternatives
- The combination of such approaches can give more complex views on network systems.
- A flexible approach is presented.
- Possible modifications – other structure of combination, other methods, dynamization of the approach, fuzzy evaluations.



Thank you for your attention!