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EVROPSKÁ UNIE

MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY

OP Vzdělávání
pro konkurenceschopnost

INVESTICE
DO ROZVOJE
VZDĚLÁVÁNÍ

Streamlining the Applied Mathematics Studies
at Faculty of Science of Palacký University in Olomouc
CZ.1.07/2.2.00/15.0243

International Conference
Olomoucian Days of Applied Mathematics

ODAM 2013

Department of Mathematical analysis
and Applications of Mathematics
Faculty of Science
Palacký Univerzity Olomouc

Vehicle routing, data uncertainty and metaheuristic algorithms

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di studi
sull'intelligenza
artificiale
USI/SUPSI
IDSIA

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Galleria 2, 6928 Manno, Switzerland

ODAM 2013

Outline

- Introduction
  - Capacitated Vehicle Routing Problem
  - Vehicle Routing Problem with Uncertain Travel Costs
  - Robust Optimization
  - Motivation of the Study
- The Solution Approach
  - The Ant Colony System
  - Robust Objective Function
- Experimental Results
  - Experimental Setup
  - The Computational Price of Robustness
  - The Operational Price of Robustness
- Conclusions

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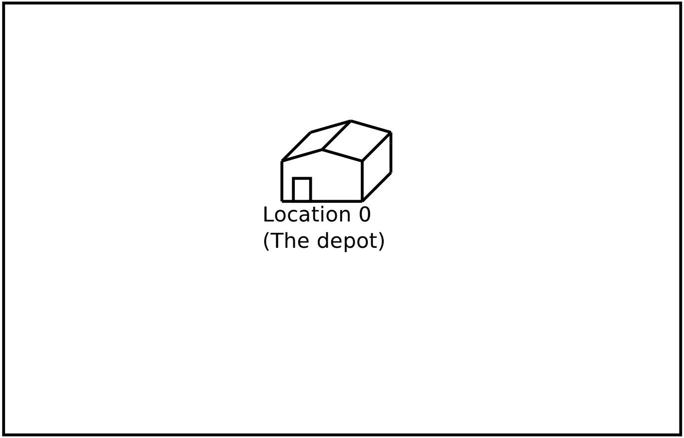
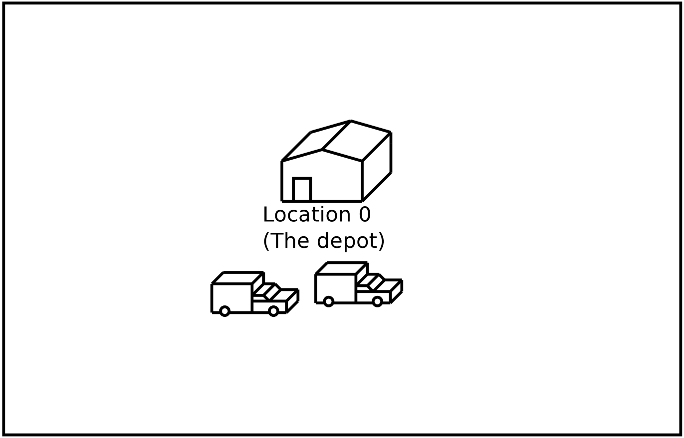


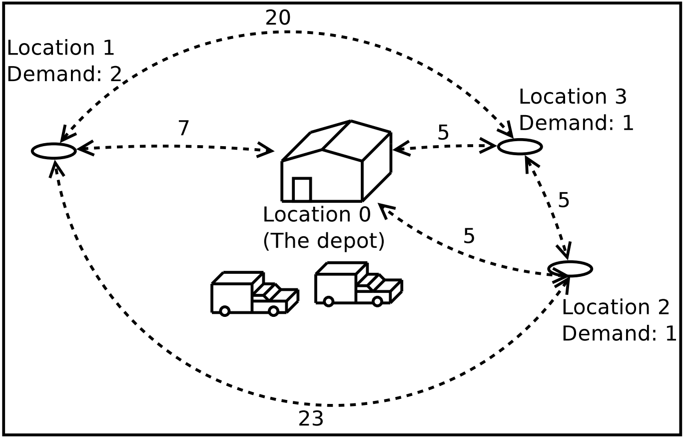
- We have a depot
-
-

Introduction

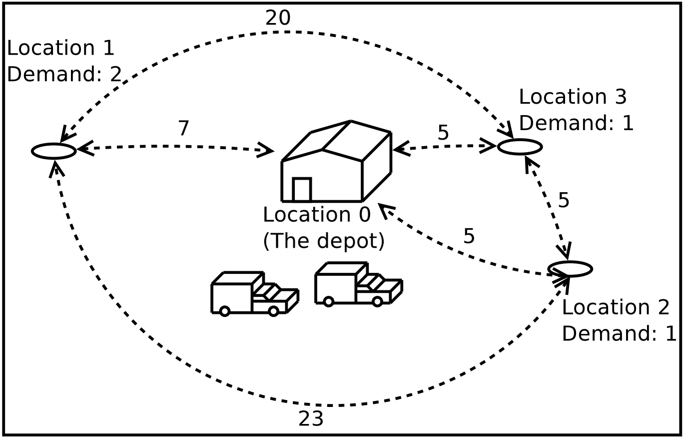
Capacitated Vehicle Routing Problem



- We have a depot
- We have a set of vehicles (with a given capacity)
-



- We have a depot
- We have a set of vehicles (with a given capacity)
- We have customer locations (with a given demand)

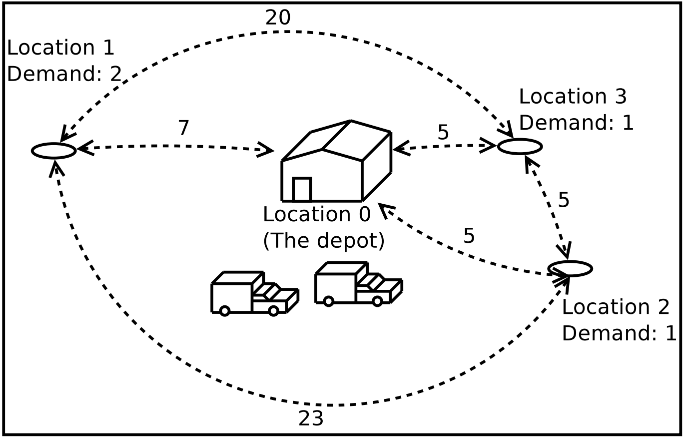


We want to select routes for vehicles, such that

- the total travel time is minimized
- all customer demands are satisfied

Introduction

Capacitated Vehicle Routing Problem



Some assumptions/constraints:

- Customers do not tolerate incomplete/partial deliveries
- We can not split deliveries (e.g. we must visit each customer once)

What is uncertainty?

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Shortly: inability to know the problem data exactly.

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Let us consider uncertainty in **travel costs**.

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Let us consider uncertainty in **travel costs**.

In the reality, it is difficult to know them exactly, because of:

- measurement errors
- unforeseen factors
  - traffic jams
  - unfriendly weather conditions

Ignoring uncertainty can lead to undesired situations

An optimal solution according to the mathematical model can turn out to be suboptimal / impractical.

Let us accept the presence of uncertainty.

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Note that no probability distribution is not known in our model.

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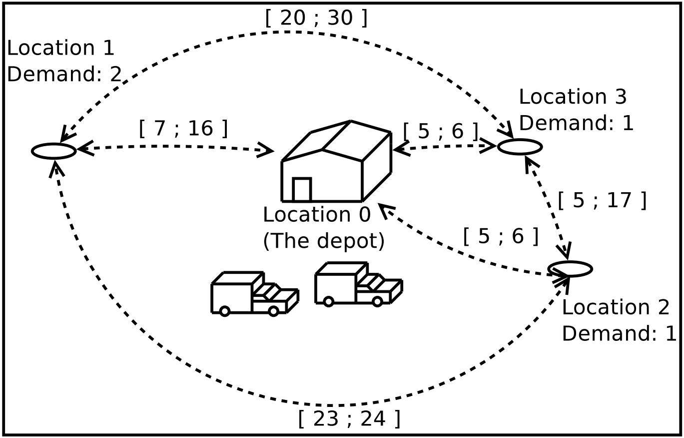
We have intervals representing possible travel costs:



What is Robust Optimization?

- Name given to methodologies which handle optimization problems with uncertain data [1] [2] [3] [4]

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[1]A.L. Soyster. Convex programming with set-inclusive constraints and applications to inexact linear programming.
Operations Research, 21(5):1154–1157, 1973

[2]L. El Ghaoui, F. Oustry, and H. Lebret. Robust solutions to uncertain semidefinite programs.
SIAM Journal on Optimization, 9(1):33–52, 1998

[3]A. Ben-Tal and A. Nemirovski. Robust solutions of uncertain linear programs.
Operations Research Letters, 25(1):1–13, 1999

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Mathematical Programming, 98(1):49–71, 2003

What is Robust Optimization?

- Name given to methodologies which handle optimization problems with uncertain data [1] [2] [3] [4]
- **Purpose:** find a practical solution which does not "go bad" because of uncertain data

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- We take a metaheuristic approach
  - to find near-optimal solutions in a practical amount of time

We incorporate **robust optimization** into an **ant colony system** metaheuristic.

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What is **Ant Colony Optimization** [5] [6] ?

- A **metaheuristic** optimization algorithm class

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What is **Ant Colony Optimization** [5] [6] ?

- A **metaheuristic** optimization algorithm class
- Developed for solving combinatorial optimization problems like **traveling salesman problem**
- Inspired by the **behavior of the ants** in the nature

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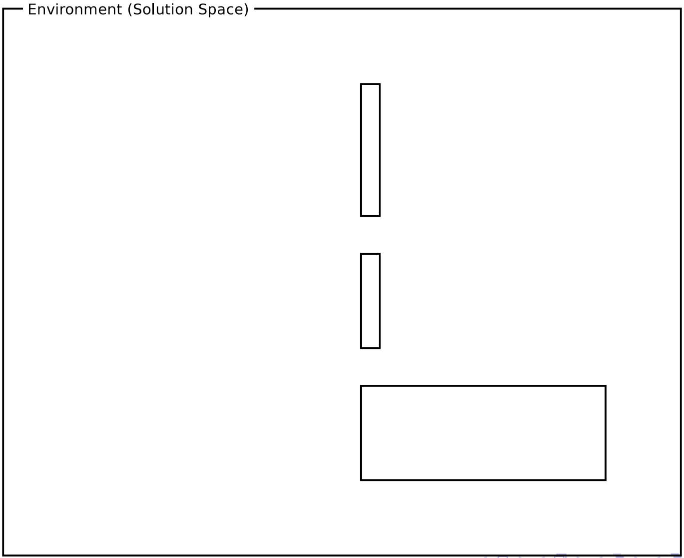
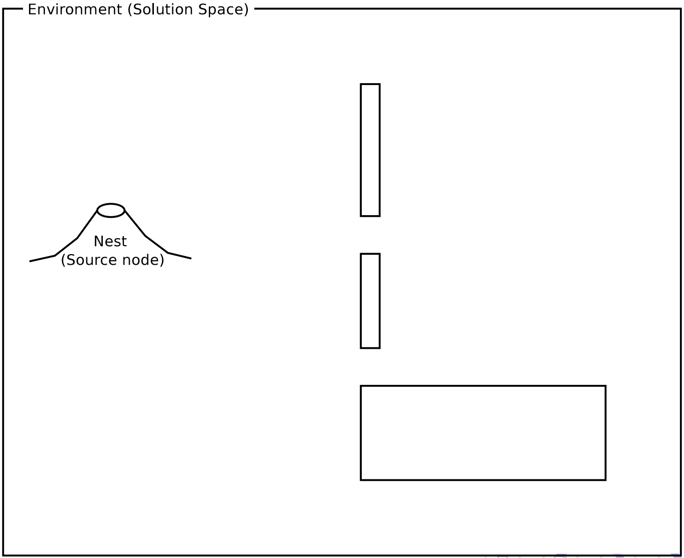

Environment (Solution Space)

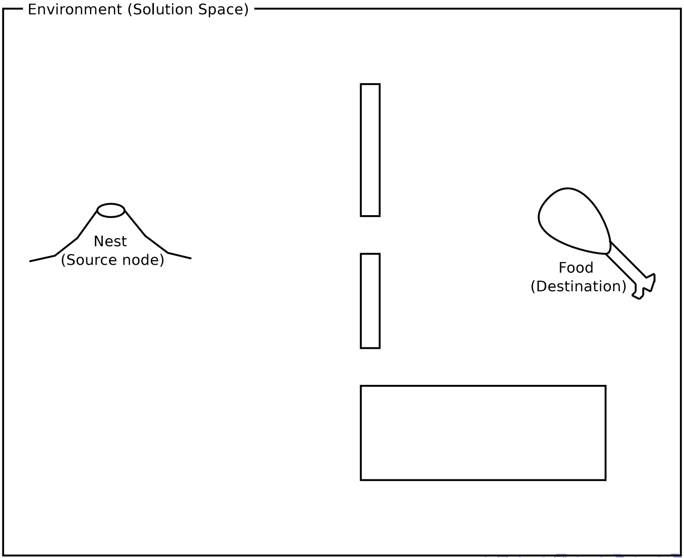
The Solution Approach

The Ant Colony System



The Solution Approach

The Ant Colony System



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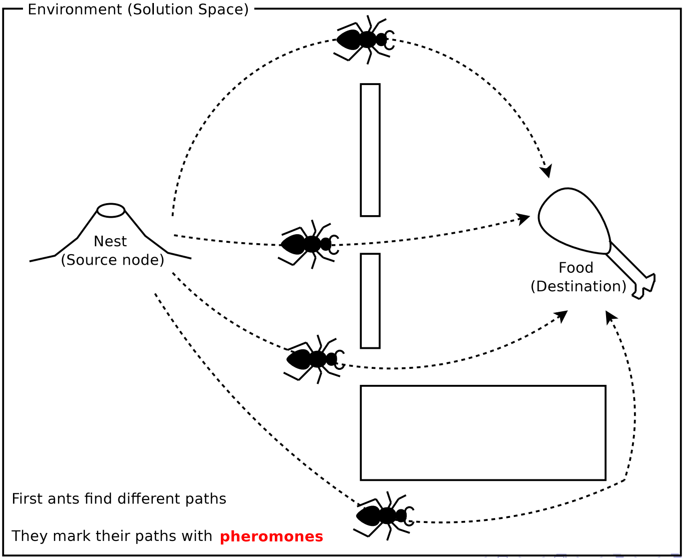

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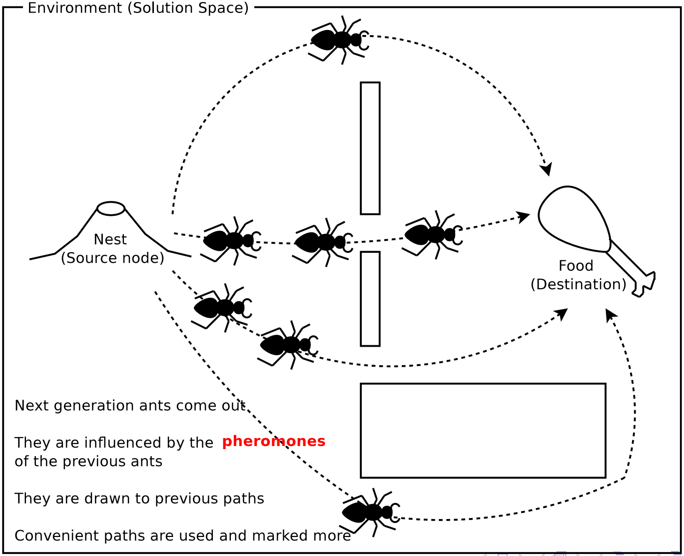

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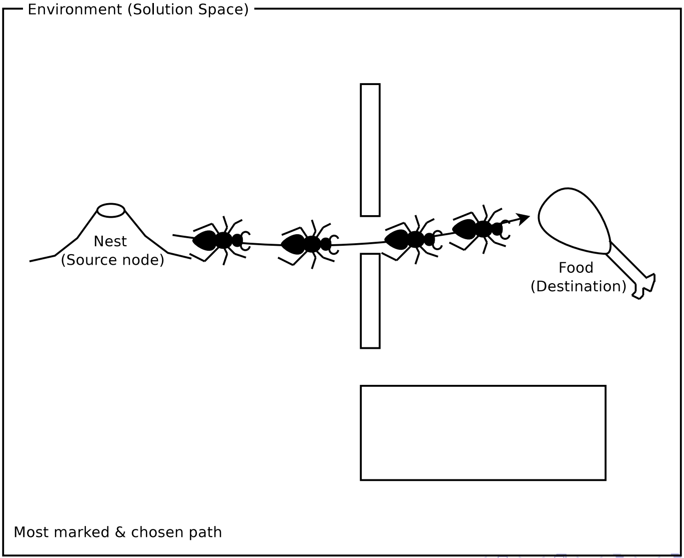


What is **ant colony system** ?

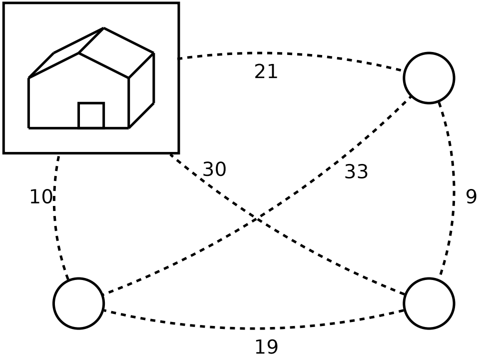
- An **ant colony optimization** variation
- Each iteration sends **multiple ants in parallel** (10 in our case)
- **Elitism**: only the best ant is allowed to put pheromones

The Solution Approach

The Ant Colony System

We simulate the ants on the graph

For simplicity, let us assume:
- There is 1 vehicle
- There is no uncertainty

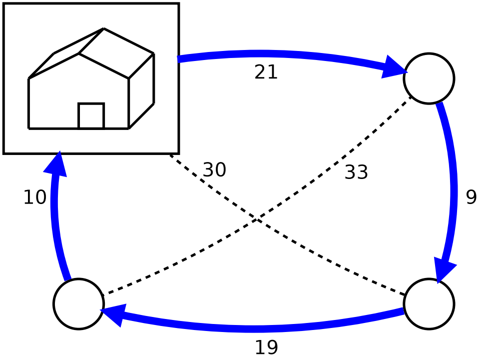


The Solution Approach

The Ant Colony System

We simulate the ants on the graph

Ant #1 generates a solution (Cost = 59)

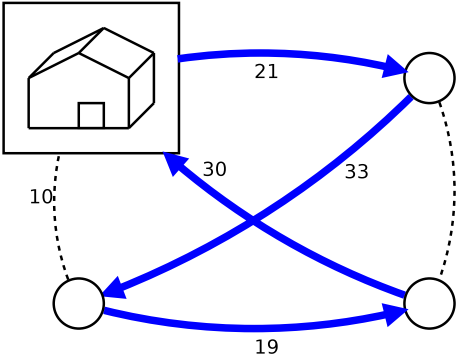


The Solution Approach

The Ant Colony System

We simulate the ants on the graph

Ant #2 generates another solution (Cost=103)

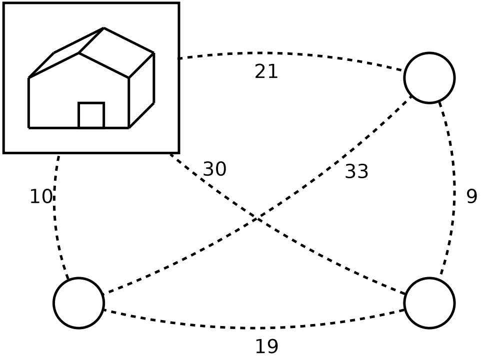


The Solution Approach

The Ant Colony System

We simulate the ants on the graph

Likewise, each ant in the colony generates its own solution

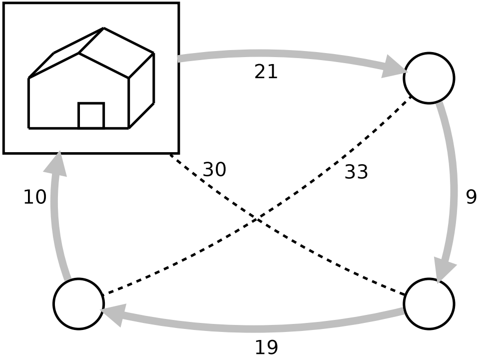


The Solution Approach

The Ant Colony System

The best solution in this iteration
was this one (Cost=59)

We leave artificial pheromones to the
arcs traveled by the best solution

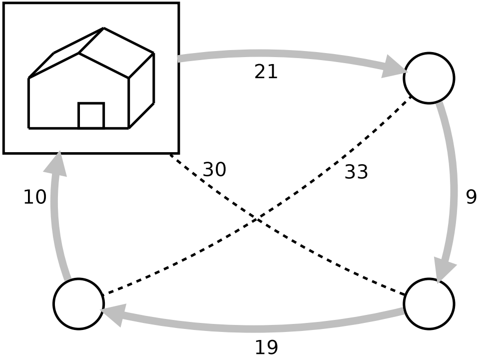


The Solution Approach

The Ant Colony System

The next iteration begins.
Each ant in the colony generate its solution.

Their choices are biased towards the arcs with more pheromones



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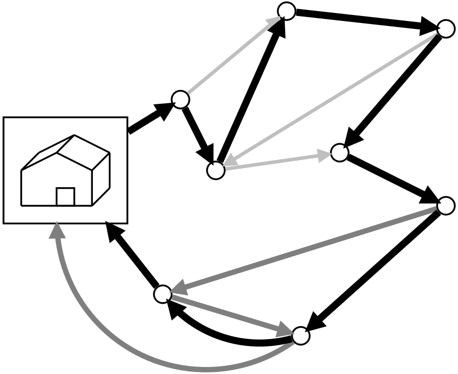

On larger instances, these can be observed:

- The best solution changes over time
- Various paths are marked with pheromones of various strengths

We use the robust optimization approach of Bertsimas & Sim [7] [8]

[7] D. Bertsimas and M. Sim. Robust discrete optimization and network flows. *Mathematical Programming*, 98(1):49–71, 2003

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Bertsimas & Sim approach:

- can be configured in terms of conservativeness
  - How much robust/protective do we want to be?

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Bertsimas & Sim approach:

- can be configured in terms of conservativeness
  - How much robust/protective do we want to be?
- computational complexity stays linear if the original problem is linear
  - Not expensive: possible to efficiently embed into a metaheuristic

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Let us suppose we have this objective function:
Minimize the cost:
$c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5$
Decision variables being $x_n \in \{0, 1\}$

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Let us now assume that there is uncertainty: $c_n \in [\underline{c}_n; \overline{c}_n]$

- If everything goes optimistically (best case): $c_n = \underline{c}_n$
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An easy-to-apply method is to be fully protective
(Soyster method [9])
Minimize $\overline{c}_1x_1 + \overline{c}_2x_2 + \overline{c}_3x_3 + \overline{c}_4x_4 + \overline{c}_5x_5$

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The Solution Approach

Robust Objective Function

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Minimize $\overline{c}_1x_1 + \overline{c}_2x_2 + \overline{c}_3x_3 + \overline{c}_4x_4 + \overline{c}_5x_5$ (over-conservative)

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If conservativeness parameter $\Gamma = 2$:

- Most dangerous 2 coefficients are assumed to be in their upper bounds
- The rest are assumed to be in their lower bounds

The Solution Approach

Robust Objective Function

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For example $\Gamma = 2$ and we have a solution χ.
If c_3 and c_5 are the two most dangerous coefficients:
$SolutionCost(\chi) = \underline{c}_1 x_1 + \underline{c}_2 x_2 + \overline{\mathbf{c}}_\mathbf{3} x_3 + \underline{c}_4 x_4 + \overline{\mathbf{c}}_\mathbf{5} x_5$

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How to calculate the cost of a solution?

Notation:

- V: set of vehicles
- sol: solution
- sol^v: set of locations visited by vehicle v
- sol_k^v: k-th visited place by vehicle v according to sol
- c_{ij}: cost of arc (i, j)

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If we had no uncertainty:

$$SolutionCost(sol) = \sum_{v \in V} \sum_{k=2}^{|sol^v|} c_{sol^v_{k-1}, sol^v_k}$$

With uncertainty:

$RobustCost(sol, \Gamma) =$

$$\max \left\{ \sum_{v \in V} \sum_{k=2}^{|sol^v|} \underline{c}_{sol^v_{k-1}, sol^v_k} + \gamma_{sol^v_{k-1}, sol^v_k} \left(\overline{c}_{sol^v_{k-1}, sol^v_k} - \underline{c}_{sol^v_{k-1}, sol^v_k} \right) \right\}$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} \gamma_{ij} \leq \Gamma$$

$$0 \leq \gamma_{ij} \leq 1 \qquad \forall (i,j) \in A$$

That is, the assumed cost is the maximum possible cost given that Γ coefficients will be maximized.

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That is, the assumed cost is the maximum possible cost given that Γ coefficients will be maximized.

Algorithmically equivalent:

– Sort all the arcs in the solution from biggest cost uncertainty to smallest cost uncertainty

– Assume that the first Γ coefficients are at their highest values

– Assume that the rest are at their lowest values

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Experimental setup:

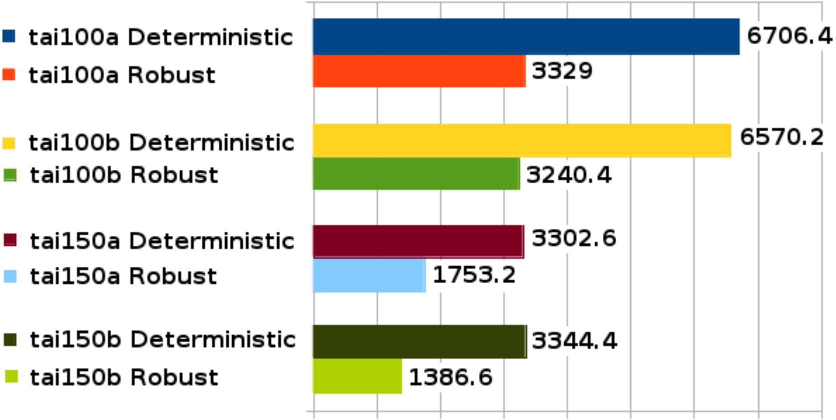
- Intel Core 2 Duo P9600 @ 2.66GHz with 4GB of RAM
- Algorithm based on the code [10], written in C
- Tried on popular instances: tai100{a,b,c,d}, tai150{a,b,c,d}
- Instances modified to have interval travel cost data

Experiments:

- Computational price of robustness
  - How slower we get when we consider the uncertainty in objective function
- Operational price of robustness
  - How is the potential cost of a solution affected by the conservativeness

[10] L.M. Gambardella, É Taillard, and G. Agazzi. *New Ideas in Optimization*, chapter "MACS-VRPTW: A Multiple Ant Colony System for Vehicle Routing Problems with Time Windows", pages 63–76. McGraw-Hill, 1999

Iterations completed within 10 seconds



The robust ant colony system is slower

- half the speed of the deterministic ant colony system

Now we analyze the effects of conservatism (Γ) on the solutions.

The analysis is done according to two axis:

- **Conservatism (Γ)**: What was the conservatism configuration during the optimization process
- **Assumption (Υ)**: How well the solution performs if we assume a real scenario where Υ number of most dangerous coefficients are perturbed towards their highest values.

Experimental Results

Operational Price of Robustness

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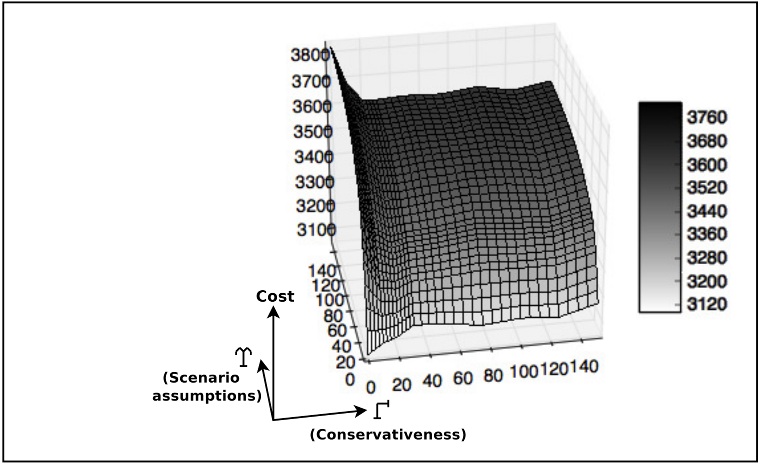
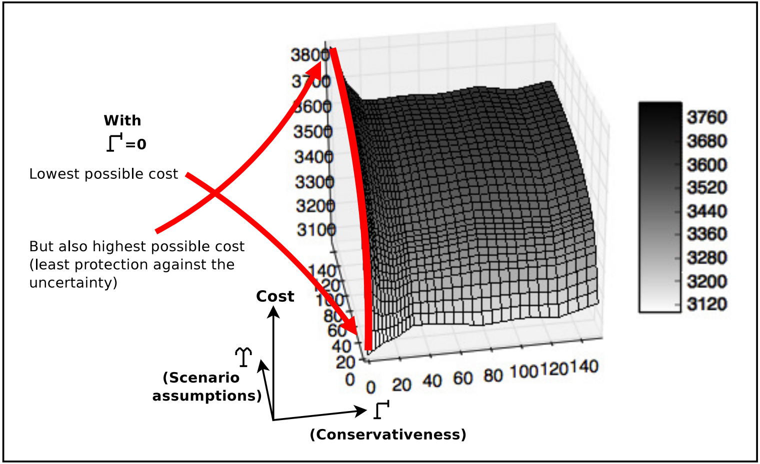
- **Conservatism (Γ)**: What was the conservatism configuration during the optimization process
- **Assumption (Υ)**: How well the solution performs if we assume a real scenario where Υ number of most dangerous coefficients are perturbed towards their highest values.

- We solved each instance with different conservatism levels to generated solution pools
- Each solving operation takes 3 minutes
- For having reliable solution pools, the best of 12 runs were taken

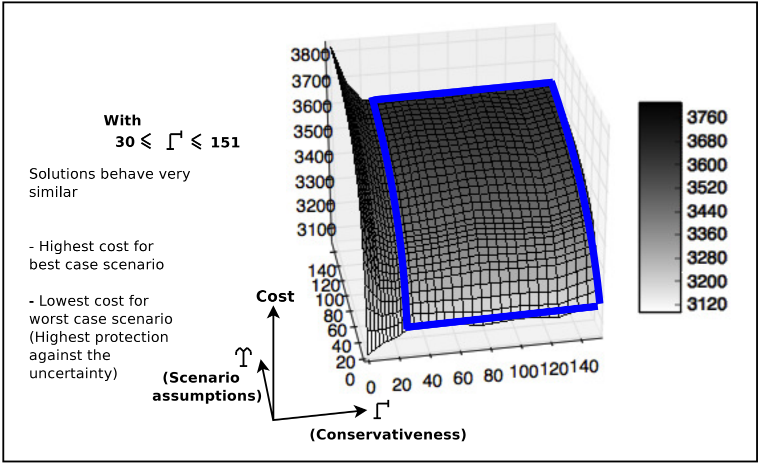
Solution pool for tai150a



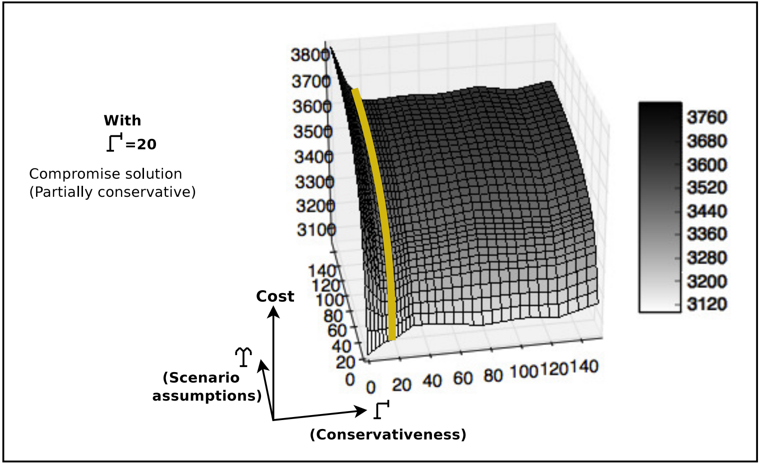
Solution pool for tai150a



Solution pool for tai150a



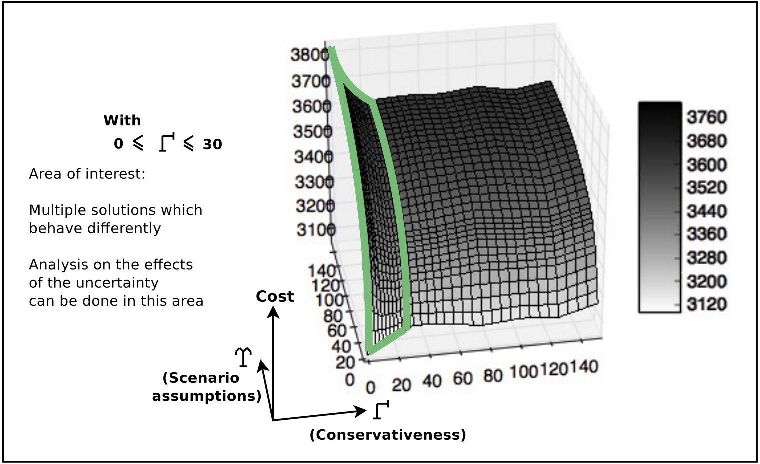
Solution pool for tai150a



Experimental Results

Operational Price of Robustness

Solution pool for tai150a



Experimental Results

Operational Price of Robustness

Similar results were found on other instances.

Instance	Solution Γ	Cost evaluations $\Upsilon=0$	$\Upsilon=10$	$\Upsilon=25$	$\Upsilon=50$	$\Upsilon=75$	$\Upsilon=101$
tai100a	0	2059.3	2262.51	2385.58	2483.72	2527.28	2542.42
	10	2067.02	2214.54	2319.65	2413.89	2464.96	2484.68
	25	2075.1	2217.22	2314.17	2407.53	2456.8	2477.0
	50	2100.52	2234.9	2332.78	2405.9	2440.32	2453.49
	75	2105.73	2229.88	2316.01	2390.74	2428.53	2444.34
	101	2110.14	2242.17	2331.53	2405.37	2440.95	2455.19
tai100b	0	2059.3	2262.51	2385.58	2483.72	2527.28	2542.42
	10	2067.02	2214.54	2319.65	2413.89	2464.96	2484.68
	25	2075.1	2217.22	2314.17	2407.53	2456.8	2477.0
	50	2100.52	2234.9	2332.78	2405.9	2440.32	2453.49
	75	2105.73	2229.88	2316.01	2390.74	2428.53	2444.34
	101	2110.14	2242.17	2331.53	2405.37	2440.95	2455.19
tai100c	0	1406.2	1574.7	1669.28	1725.35	1750.76	1762.02
	10	1421.6	1540.59	1623.88	1676.82	1700.11	1710.43
	25	1442.17	1539.99	1603.83	1657.4	1682.63	1694.73
	50	1463.51	1564.07	1621.19	1665.74	1687.35	1697.2
	75	1447.11	1557.79	1620.49	1668.53	1694.19	1707.81
	101	1463.25	1554.89	1606.65	1654.52	1677.93	1688.95
tai100d	0	1596.31	1736.59	1835.16	1921.65	1969.65	1986.28
	10	1607.26	1721.96	1812.32	1904.64	1954.99	1973.84
	25	1604.11	1712.9	1802.47	1888.03	1932.95	1948.57
	50	1629.59	1737.5	1810.49	1884.54	1927.25	1943.91
	75	1606.7	1728.08	1817.47	1894.48	1930.1	1944.75
	101	1668.21	1750.43	1821.35	1887.88	1925.2	1938.9

Similar results were found on other instances.

Instance	Solution Γ	Cost evaluations $\Upsilon=0$	$\Upsilon=10$	$\Upsilon=20$	$\Upsilon=30$	$\Upsilon=50$	$\Upsilon=75$	$\Upsilon=100$	$\Upsilon=125$	$\Upsilon=151$
tai150a	0	3057.94	3390.46	3538.96	3615.69	3707.36	3763.75	3797.9	3815.98	3825.48
	10	3097.5	3277.93	3382.75	3450.47	3542.32	3609.26	3648.49	3669.62	3679.91
	20	3118.97	3270.62	3353.47	3411.85	3492.84	3547.98	3584.12	3604.21	3614.06
	30	3156.47	3294.29	3371.55	3423.79	3490.68	3543.25	3579.6	3598.69	3608.56
	50	3149.19	3298.85	3382.76	3433.91	3502.35	3555.47	3589.55	3607.79	3615.42
	75	3129.4	3310.89	3400.62	3453.07	3517.03	3562.12	3587.66	3602.02	3610.41
	100	3142.33	3305.57	3386.24	3441.66	3514.01	3564.67	3597.65	3616.71	3627.67
	125	3131.15	3304.53	3384.42	3432.99	3495.98	3544.47	3576.53	3593.37	3602.18
	151	3171.07	3340.74	3424.33	3470.37	3528.64	3573.75	3601.08	3617.73	3626.56
tai150b	0	2739.21	3069.54	3198.94	3274.44	3359.1	3418.57	3453.35	3474.16	3484.27
	10	2766.41	2921.46	2986.57	3028.48	3085.07	3131.84	3161.56	3179.8	3189.79
	20	2832.26	2948.69	3000.81	3039.63	3096.09	3141.99	3170.57	3186.99	3195.47
	30	2828.24	2931.2	2985.24	3029.07	3091.45	3142.14	3173.95	3193.3	3202.86
	50	2778.8	2904.21	2955.55	2992.29	3044.94	3091.15	3120.9	3137.98	3146.97
	75	2799.36	2919.01	2973.59	3013.9	3070.32	3118.89	3151.56	3171.03	3180.76
	100	2825.5	2925.73	2979.14	3016.48	3072.69	3123.69	3156.0	3175.6	3185.57
	125	2792.57	2912.71	2962.05	2998.15	3050.61	3096.77	3127.12	3144.79	3153.86
	151	2845.26	2957.83	3021.25	3061.43	3110.56	3148.17	3172.65	3188.03	3196.82
tai150c	0	2424.0	2710.5	2835.77	2891.73	2948.93	2991.88	3019.11	3035.43	3044.21
	10	2498.13	2657.34	2751.06	2811.87	2878.65	2925.59	2952.89	2969.05	2976.76
	20	2524.18	2671.85	2719.62	2752.32	2799.04	2839.41	2867.92	2885.6	2894.25
	30	2484.0	2629.85	2671.19	2703.18	2752.03	2795.3	2824.23	2841.68	2851.8
	50	2508.54	2632.85	2695.3	2733.76	2780.41	2818.37	2840.73	2855.34	2862.65
	75	2469.62	2577.21	2626.77	2658.43	2703.07	2741.03	2765.74	2780.03	2787.32
	100	2459.48	2616.64	2688.32	2730.48	2788.26	2831.81	2858.76	2874.55	2881.33
	125	2515.31	2639.38	2691.0	2724.58	2771.67	2809.58	2833.72	2849.56	2856.55
	151	2532.29	2657.48	2702.28	2732.72	2778.58	2818.02	2842.06	2858.33	2867.75
tai150d	0	2662.84	2932.68	3075.43	3143.09	3226.07	3280.73	3311.14	3326.35	3333.13
	10	2700.91	2884.32	2983.02	3050.67	3133.85	3192.25	3224.2	3242.3	3251.39
	20	2750.85	2913.03	2987.75	3043.98	3118.76	3168.32	3196.22	3212.07	3219.87
	30	2768.92	2903.33	2971.89	3020.14	3088.88	3143.1	3178.74	3198.66	3207.74
	50	2739.32	2889.63	2949.71	2994.59	3054.79	3102.39	3131.79	3148.18	3156.05
	75	2809.45	2924.43	2986.61	3024.55	3082.63	3132.18	3163.69	3182.11	3190.63
	100	2737.14	2929.63	2997.12	3035.84	3089.46	3132.93	3157.66	3172.01	3179.5
	125	2754.36	2915.53	2979.61	3027.07	3089.16	3136.65	3164.26	3179.6	3186.36
	151	2776.35	2928.92	2993.73	3033.26	3084.55	3123.54	3148.25	3163.04	3169.71

Outline

Conclusions

Some conclusions:

- An ant colony system was improved to consider uncertainty
- The robust version of ant colony system now allows the decision maker to generate a solution pool containing solutions of different conservativeness levels

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- An ant colony system was improved to consider uncertainty
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Future work:

- Handle time window constraints with uncertainty considerations

An Ant Colony System for the Capacitated Vehicle Routing Problem with Uncertain Travel Costs

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and Luca Maria Gambardella)
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Thank you for your attention!