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EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

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Faculty of Science
Palacký Univerzity Olomouc

Computational Intelligence for Image Processing

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OP Výzkum
a vývoj pro inovace

Outline

Outline

Image Processing in Problems

- ✓ Representation
- Quality Improving (contrast enhancement, sharpening, etc.)
- ✓ Edge Detection
- Compression (Reduction) and Reconstruction
- ✓ Reconstruction of Damages
- Fusion
- Segmentation
- ✓ Registration

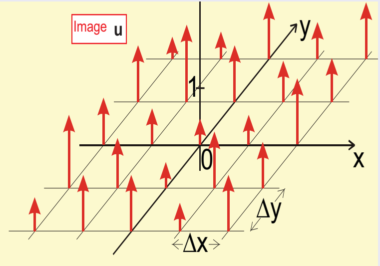
Computation Intelligence. Why?

- To bring intuition in a form of knowledge
- To keep connection between a problem and a method of solving
- To explain "**why it works**"

Image as an Object

Image

2D gray-scaled image **u** : $D \longrightarrow R$ is identified with **intensity function** of two variables such that the domain D is sampled, and the range R is quantized.



Domain -
$D = \{1, \ldots, N\} \times \{1, \ldots, M\}$
Range -
$R = \{0, 1, \ldots, 255\}$
Pixel -
image sample.

Representations of Image

Image

$$\mathbf{u} : \{1,\dots,N\} \times \{1,\dots,M\} \longrightarrow \{0,1,\dots,255\},$$

can be represented by:

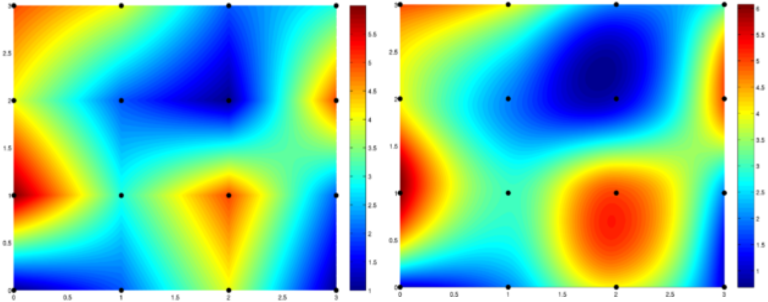
- *$N \times M$ matrix* $\mathbf{U} = (u_{ij})$ where $u_{ij} = \mathbf{u}(i,j)$
- *interpolating (approximating)* function

$$\hat{\mathbf{u}} : [1,N] \times [1,M] \longrightarrow [0,255],$$

such that for all $i = 1,2,\dots,N$, $j = 1,2,\dots,M$,

$$\mathbf{u}(i,j) = \hat{\mathbf{u}}(i,j) \quad (\mathbf{u}(i,j) \approx \hat{\mathbf{u}}(i,j)).$$

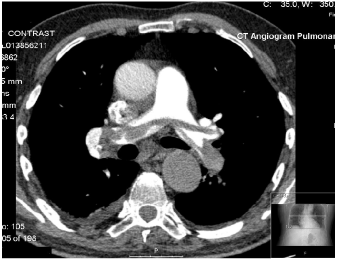
Representations by Interpolation

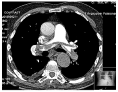
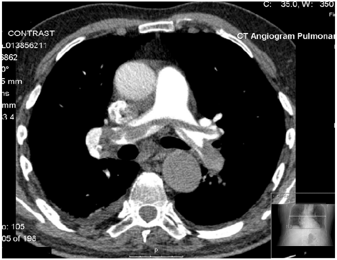


Bilinear (left) and bicubic interpolations as **magnifiers**.

Approximate Representation by F-transform

$$\mathbf{u} \Rightarrow F[\mathbf{u}] \Rightarrow \hat{\mathbf{u}}$$



$\mathbf{u}$ and $F[\mathbf{u}]$ are represented by matrices, $\hat{\mathbf{u}}$ – analytically.

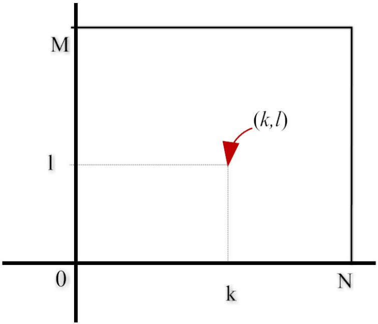
Outline

F-transform

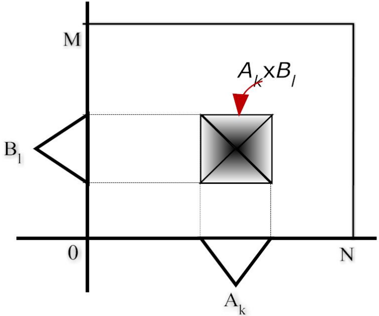
F-transform – result of a weighted projection of an image on a finite set of granules that establish a fuzzy partition of a domain.

Granulation of Domain $[1, N] \times [1, M]$

Rectangular area $[1, N] \times [1, M]$

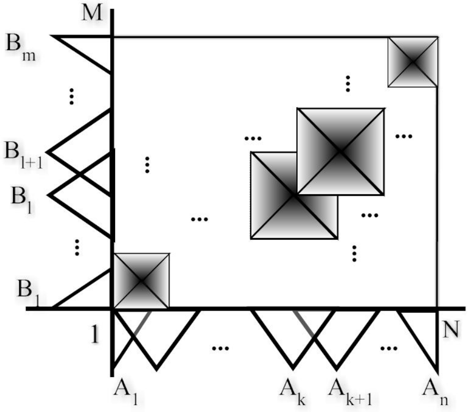


Granule $A_k \times B_l$



Fuzzy sets $A_k : [1, N] \longrightarrow [0, 1]$, $B_l : [1, M] \longrightarrow [0, 1]$.

Fuzzy Partition of $[1, N] \times [1, M]$



Granules $A_k \times B_l$, $k = 1, \ldots, n$, $l = 1, \ldots, m$, are associated with **selected nodes** (k, l), $3 \leq k \leq N$, $3 \leq l \leq M$.

Matrix Representation of Fuzzy Partition of $[1, N]$

- $A_1, \ldots, A_n$ – **fuzzy partition** of $[1, N]$
- Denote $a_{ki} = A_k(i)$, $a_k = \sum_{i=1}^{N} a_{ki}$, $a^i = \sum_{k=1}^{n} a_{ki}$, $k = 1, \ldots, n$, $i = 1, \ldots, N$
- $A_{n \times N} = \begin{pmatrix} a_{11} & \ldots & a_{1N} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{nN} \end{pmatrix}$ – matrix of fuzzy partition
- $P(A)_{n \times n} = \begin{pmatrix} \frac{1}{a_1} & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & \frac{1}{a_n} \end{pmatrix}$ – raw normalization
- $Q(A)_{N \times N} = \begin{pmatrix} \frac{1}{a^1} & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & \frac{1}{a^N} \end{pmatrix}$ – column normalization

Direct F-transform of image u (Reduction)

- $\mathbf{u}$ is represented by matrix $\mathbf{U}_{N\times M} = (u_{ij})$ where $u_{ij} = \mathbf{u}(i,j)$
- fuzzy partition $A_1, \ldots, A_n$ – by matrix $A_{n\times N}$, $n < N$
- fuzzy partition $B_1, \ldots, B_m$ – by matrix $B_{m\times M}$, $m < M$

The $n \times m$ matrix $F[\mathbf{u}]$ is the direct **F-transform** of $\mathbf{u}$ w.r.t. $A_1 \times B_1, \ldots, A_n \times B_m$ where

$$F[\mathbf{u}]_{n\times m} = (P(A)A)\mathbf{U}_{N\times M}(P(B)B)^T$$

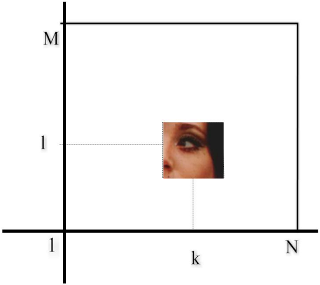
and the components are

$$F[\mathbf{u}]_{kl} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\mathbf{u}(i,j)A_k(i)B_l(j)}{\sum_{i=1}^{N}\sum_{j=1}^{M}A_k(i)B_l(j)}.$$

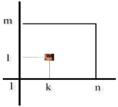
F-transform as a Weighted Projection

Image u

Image block on $A_k \times B_l$



$F[\mathbf{u}]_{kl}$



Inverse F-transform of image u (Magnification)

The $N \times M$ matrix $\hat{\mathbf{u}}$ is the representation of the inverse **F-transform** of $\mathbf{u}$ on $\{1, \ldots, N\} \times \{1, \ldots, M\}$ where

$$\hat{\mathbf{u}}_{N \times M} = (AQ(A))^T F[\mathbf{u}]_{n \times m} (BQ(B))$$

and the pixels are as follows

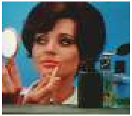
$$\hat{\mathbf{u}}(i,j) = \frac{\sum_{k=1}^{n} \sum_{l=1}^{m} F[\mathbf{u}]_{kl} A_k(i) B_l(j)}{\sum_{k=1}^{n} \sum_{l=1}^{m} A_k(i) B_l(j)},$$

$i = 1, \ldots, N, j = 1, \ldots, M.$

Inverse F-transform as Magnifier

$\hat{\mathbf{u}}$ – inverse F-transform of $\mathbf{u}$

$F[\mathbf{u}]$ – F-transform of $\mathbf{u}$



Main Properties of F-Transform

Best Approximation

Component $F[\mathbf{u}]_{kl}$, $k = 1, \ldots, n$, $l = 1, \ldots, m$, **minimizes the following criterion**

$$\sum_{i=1}^{N} \sum_{j=1}^{M} (\mathbf{u}(i,j) - x)^2 A_k(i) B_l(j)$$

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Main Properties of F-Transform

F-Transform of Constants

Components of a **constant image** $\mathbf{u}$ where $\mathbf{u}(i, j) = c$ coincide with c, i.e. for all $k = 1, \ldots, n, l = 1, \ldots, m$,

$$F[\mathbf{u}]_{kl} = c.$$

Main Properties of F-Transform

Linearity

The map $\mathbf{F} : \mathbf{u} \mapsto F[\mathbf{u}]$ is **linear**, i.e. for all $\mathbf{u}, \mathbf{v}$, and for all $\alpha, \beta \in \mathbb{R}$,

$$\mathbf{F}(\alpha\mathbf{u} + \beta\mathbf{v}) = \alpha\mathbf{F}(\mathbf{u}) + \beta\mathbf{F}(\mathbf{v}).$$

Outline

F-Transform for Image Reduction

Image Reduction aims at adjusting image to a smaller screen.

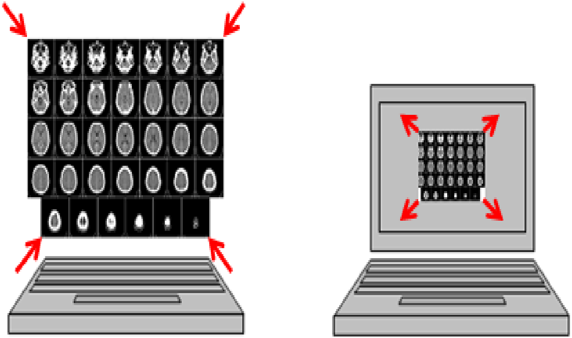


Image Reduction in Examples, Ratio $= 1/9$

The image reduction is performed by the **direct F-transform**.

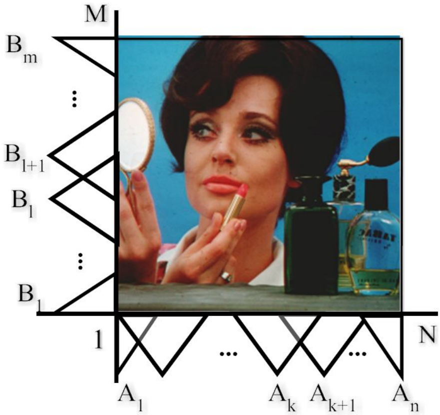


Image Reduction in Examples, Ratio $= 1/9$

Filtering by Reduction

Original image contains 20% of impulsive noise

MSE-quality of Reductions: FT versus Interpolation

Various reduction algorithms are compared on a set that contains 53 color images from http://sipi.usc.edu/database/database.php/volume=textures.

Type	FT	Bilinear	Bicubic	Lanczos
Min	**32.6**	43.6	44.7	41.9
1st Qu.	**81.2**	113.7	113.8	121.4
Median	**102.1**	145.6	150.0	170.8
Mean	**146.5**	197.3	199.6	212.0
3rd Qu.	**163.0**	246.3	242.0	262.6
Max	**517.0**	606	607.0	626.7

All statistics show the advantage of the FT-reduction !

Outline

Image Reconstruction

Image reconstruction takes a damaged image (noise, text, scratch, etc.) and restores it.

Assumption. The damaged area must be separated !

Image Reconstruction. Illustration

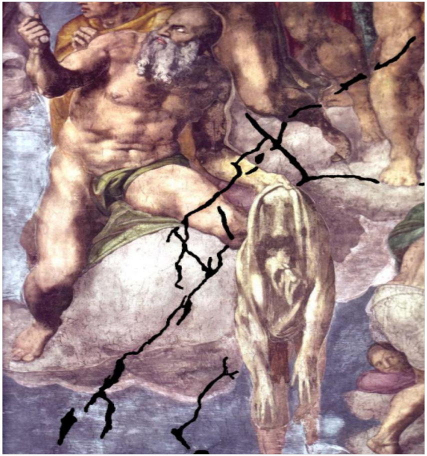


Image Reconstruction. Illustration

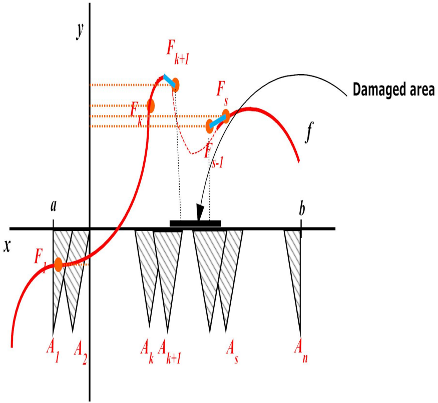
F-transform for Image Reconstruction

Proposed Method

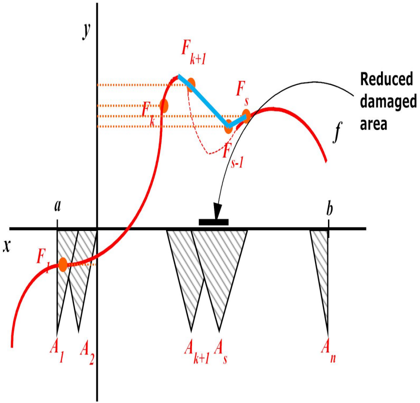
Repeat until the image is fully reconstructed

- choose a fuzzy partition and compute the respective inverse F-transform;
- if a pixel of the original image is undamaged then it is chosen for the reconstructed image, otherwise it is replaced by the value of the inverse F-transform.

F-transform Reconstruction in steps

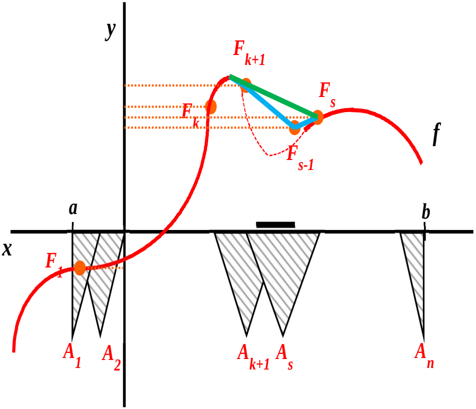


"Narrow" granules in partition



"Wide" granules in partition

F-transform versus Interpolation



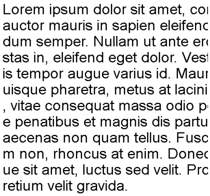
- Bilinear interpolation
- F-transform

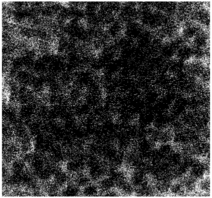
Illustration of the Method

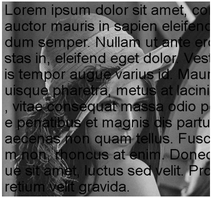
RMSD = 4.56

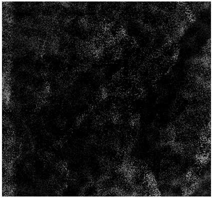
Illustration of the Results

Types of damage









Impaint **Text** **70% Noise**

Illustration of the Results

Reconstruction

MSE-quality of Reconstructions: FT versus Interpolation

Damage	**RBF**	**FT**
Noise	12.47	11.72
Inpaint	5.23	4.87
Text	4.62	4.56

Outline

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"Edge" as a Notion

- "Edge" is an area where image intensity **abruptly** changes.
- Edge detection is aiming at searching **local maxima** of the gradient magnitude.

F^1-transform

- Let f be a differentiable function on $[a, b]$.
- The F-transform (with constant components) can be generalized to the **F^1-transform with linear components:**

$$F^1[f] = [F_1^1, \dots, F_n^1]$$

where for each $k = 1, \dots, n$

$$F_k^1 = c_{k,0}\ P_k^0 + c_{k,1}\ P_k^1$$

F^1-transform and Orthogonal Projections

- Every component

$$F_k^1 = c_{k,0}\, P_k^0 + c_{k,1}\, P_k^1, \quad k = 1, \ldots, n,$$

is a **weighted orthogonal projection** of f onto a linear subspace of functions with the basis of polynomials $P_k^0 = 1, P_k^1 = (x - x_k)$,

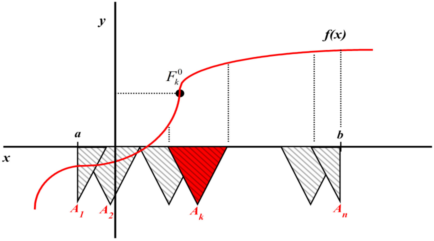
$$F_k^1 = c_{k,0} + c_{k,1}\,(x - x_k),$$

where

$$c_{k,0} = F[f]_k, \quad c_{k,1} \approx f'(x_k).$$

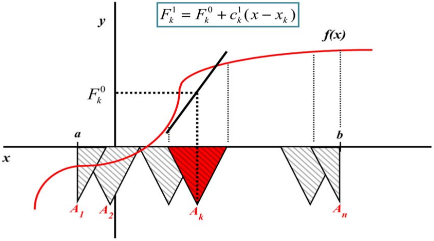
F-transform and F^1-transform

F-transform



$$F_k = F_k^0 = \frac{\int_a^b (f(x)\bullet 1) A_k(x)dx}{\int_a^b A_k(x)dx}$$

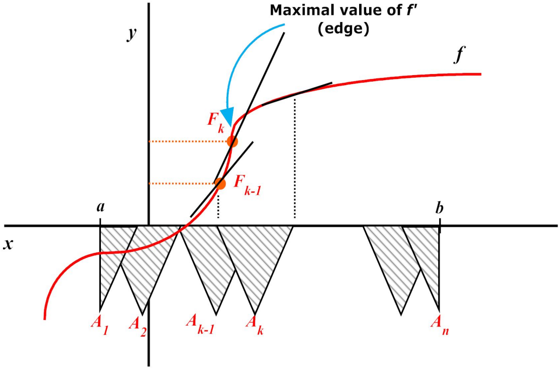
F^1-transform



$$c_k^1 = \frac{\int_a^b f(x)\bullet(x - x_k) A_k(x)dx}{\int_a^b (x - x_k)^2 A_k(x)dx}$$

F^1-transform for Edge Localization

Edge as a local maximum of a derivative magnitude



Approximation of the gradient

- Gradient vector $(\mathbf{u}_x, \mathbf{u}_y)$ of the image function $\mathbf{u}$ at (k, l) is approximated by $(c^1_{kl,1}, c^1_{kl,2})$ – vector of coefficients of the respective linear component of F^1-transform of $\mathbf{u}$:

$$F^1(x, y) = c^1_{kl,0} + c^1_{kl,1}(x - k) + c^1_{kl,2}(y - l),$$

$k = 1, \ldots, n,\ l = 1, \ldots, m$.

F^1-transform edge detection

Original image

F^1-transform detected edges

Original image - Lena

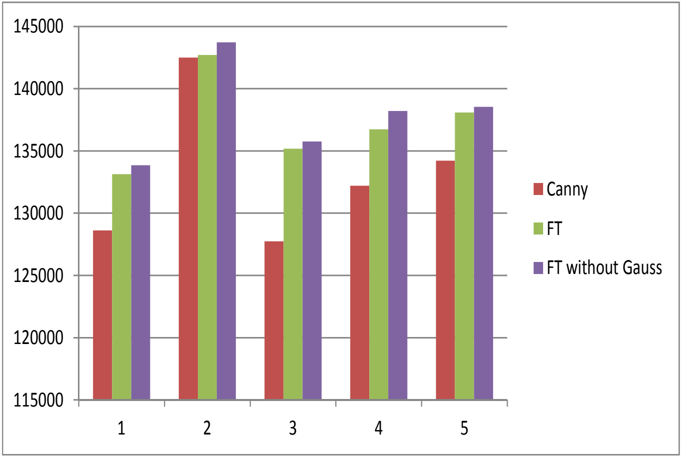
Blurred background, texture on the hat, perfect main contours, noise.

Comparison - Lena

“Classical” Canny algorithm

F^1-transform edge detector

Number of Detected Edges



Outline

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Compression

Original “Cameraman”

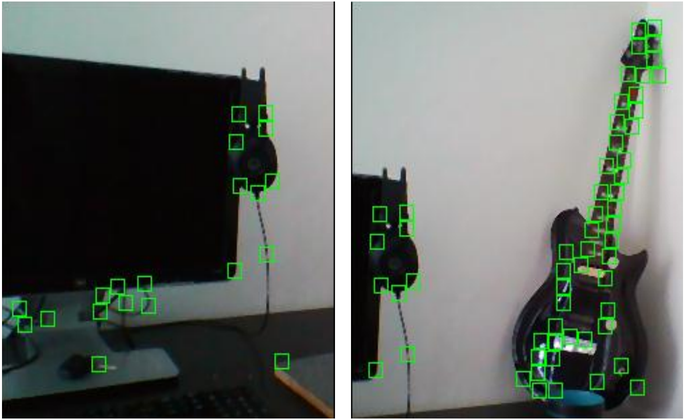
Compression

F-transform reconstructed

left CR=0.25, PSNR=37 (JPEG 39); **right** CR=0.44, PSNR=43 (46)

Registration

Registration



Registration

Outline

Conclusion

- F-transform makes manipulating with objects in a target domain much easier
- FT reduction is fast and efficient
- FT reconstruction is easier than the interpolation based reconstruction
- FT edge detection has better quality than Canny's detector

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